(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2016/2017 ACADEMIC YEAR

SECOND YEAR FIRST SEMESTER

SUPPLEMENTARY/SPECIAL EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

CHEMISTRY

COURSE CODE: MAT 214

COURSE TITLE: VECTOR ANALYSIS

DATE: 26/09/17 **TIME:** 8AM -10AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE COMPULSORY (30 MARKS)

a) Define the following terms (2 mks)
   (i) Vector field
   (ii) Scalar field

b) Given that $\vec{A} = 4i - 3j + 2k$ and $\vec{B} = i - 2j + 4k$ find
   (i) $\vec{A}.\vec{B}$
   (ii) $|\vec{A} + \vec{B}|$
   (iii) The angle between $\vec{A}$ and $\vec{B}$ (6 mks)

c) If $\vec{P} = 2i - 3j + k$ and $\vec{Q} = i + 4j - 2k$ find
   (i) $\vec{P} \times \vec{Q}$
   (ii) $(\vec{P} + \vec{Q}) \times (\vec{P} - \vec{Q})$ (6 mks)

d) If $\vec{V} = (x^2 - y^2)i + 2xyj + (y^2 - xy)k$ find
   (i) Divergence of $\vec{V}$
   (ii) Curl of $\vec{V}$ (5 mks)

e) Prove that $\nabla \times (\vec{A} + \vec{B}) = \nabla\vec{A} + \nabla\vec{B}$ (4 mks)

f) Find the total work done in moving a particle in a force field given by $\vec{F} = 3xyj - 5zj + 10xk$ along the curve $x = t^2$, $y = 2t^2$ and $z = t^3$ From $t = 1$ to $t = 2$ (5 mks)

g) State the Green's theorem (2 mks)

QUESTION TWO (20 MARKS)

a) Given that $\vec{A} = 3i - j + k$ and $\vec{B} = 2i + 3j - k$ determine the vector perpendicular to the plane of $\vec{A}$ and $\vec{B}$ (4 mks)

b) Find the unit normal to the surface $x^2y + 2xz = 4$ at $(2, -2, 3)$ (3 mks)

c) Find the directional derivative of $\varphi = x^2yz + 4xz$ at $(1, -2, -1)$ in the direction of $2i - j - 2k$ (4 mks)

d) If $\varphi = 2xyz^2$ and C is the curve, $x = t^2$, $y = 2t$, $z = t^3$ from $t = 0$ to $t = 1$ evaluate the line integral $\oint \varphi dr$ (3 mks)

e) If $\vec{F}(t) = (3t^2 - t)i + (2 - 6t)j - 4tk$ find $\int_2^4 \vec{F}(t)dt$ (2 mks)

f) A particle moves along the curve whose parametric equations are given by

   $x = 2t^2$, $y = t^2 - 4t$ and $z = 3t - 5$ where t is time in seconds.

   Find the components of its velocity and acceleration at the time $t = 1$

   in the direction $2i - j + k$ (4 mks)

QUESTION THREE (20 MARKS)

a) If $\vec{A} = (3x^3y + 2x^3)i + (e^{xy} - y\cos 3x)j - \sin yk$ find

(i) $\frac{\partial^2 \vec{A}}{\partial x^2}$

(ii) $\frac{\partial^2 \vec{A}}{\partial y^2}$ (4 mks)

b) Find the projection of vector that $\vec{A} = 4i - 4j + 7k$ on the vector $\vec{B} = i - 2j + k$ (3 mks)

c) If that $\vec{A} = 4t^2i - tj + t^3k$ and $\vec{B} = \sin ti - \cos tj$ find

(i) $\frac{d}{dt}(\vec{A}.\vec{B})$

(ii) $\frac{d}{dt}(\vec{A} \times \vec{B})$

(iii) $\frac{d}{dt}(\vec{A}.\vec{A})$ (6 mks)

d) If $\vec{A} = z^3xi - 2x^2yzj + 2yz^4k$ find $\nabla.(\nabla \times \vec{A})$ (3 mks)

e) Given that vectors $a^l = \frac{b \times c}{a.b \times c}$, $b^l = \frac{c \times a}{a.b \times c}$ and $c^l = \frac{a \times b}{a.b \times c}$ show that if $a.b \times c \neq 0$

(i) $a^l.a = b^l.b = c^l.c = 1$

(ii) $a^l b = 0$ (4 mks)

QUESTION FOUR (20 MARKS)

a) State the Stokes theorem (2 mks)

b) Determine the constant λ so that the vector $\vec{V} = (x + 3y)i + (y - 2z)j + (x + \lambda z)k$ is Solenoidal (3 mks)

c) If $\vec{A} = xz^3i - 2x^2yzj + 2yz^4k$ find Curl $\vec{A}$ at the point (1,-1.1) (3 mks)

d) If $\vec{A} = \cos xyi + (3xy - 2x^2)j - (3x + 2y)k$ find $\frac{\partial^2}{\partial x \partial y}(\vec{A})$ at (1,0,-2) (3 mks)

e) Find the area of a triangle having vertices at P(1,3,2), Q(2,-1,1) and R(-1,2,3) (4 mks)

f) Prove that $\nabla^2\left(\frac{1}{r}\right) = 0$ where $r = \sqrt{x^2 + y^2 + z^2}$ and $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ (5 mks)

QUESTION FIVE (20 MARKS)

a) State the Gauss Divergence theorem (2 mks)

b) Evaluate $\iint \vec{A}\tilde{n}ds$ where $\vec{A} = 18zi - 12j + 3yk$ and S is that part of the plane $2x + 3y + 6z = 12$ which is located in the first octant (8 mks)

c) Evaluate the line integral given $\vec{P} = (3x^2 + 6y)i - 14yzj + 20xz^2k$ From (0,0,0) to (1,1,1) along the path C (i) $x = t, y = t^2$ and $z = t^3$ (ii) the straight lines from (0,0,0) to (1,0,0) then to (1,1,0) and then (1,1,1) (iii) the straight line joining (0,0,0) to (1,1,1) (10 mks)