



*(Knowledge for Development)*

**KIBABII UNIVERSITY**  
**UNIVERSITY EXAMINATIONS**  
**2017/2018 ACADEMIC YEAR**  
**SECOND YEAR FIRST SEMESTER**  
**MAIN EXAMINATION**  
**FOR THE DEGREE OF BACHELOR OF SCIENCE**  
**CHEMISTRY**

**COURSE CODE:** MAT 214

**COURSE TITLE:** VECTOR ANALYSIS

**DATE:** 16/01/18

**TIME:** 2 PM -4 PM

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**INSTRUCTIONS TO CANDIDATES**

Answer Question One and Any other TWO Questions

TIME: 2 Hours

### QUESTION ONE (30 MARKS)

- (a) Given that  $\mathbf{A} = 2\mathbf{i} - 3\mathbf{j} - \mathbf{k}$ , and  $\mathbf{B} = \mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$ , show that  $\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A}$ . (5mks)
- (b) Find the unit vector parallel to the resultant of the vectors  $\mathbf{r}_1 = 2\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}$ ,  $\mathbf{r}_2 = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ ; Resultant vector  $\mathbf{R} = \mathbf{r}_1 + \mathbf{r}_2$ . (5mks)
- (c) Evaluate the directional derivative of  $\varphi = x^2yz + 4xz^2$  at  $(1, -2, -1)$  in the direction  $2\mathbf{i} - \mathbf{j} - 2\mathbf{k}$ . (5mks)
- (d) Determine the divergence and curl of the vector field  $\mathbf{F} = x\mathbf{i} + y\mathbf{j}$ . (5mks)
- (e) Convert  $(1, \sqrt{3}, 2)$  to spherical coordinate system. (3mks)
- (f) Convert  $(\sqrt{6}, \frac{\pi}{4}, \sqrt{2})$  to spherical coordinate system. (3mks)
- (g) State the following theorems.
- (i) Stokes theorem. (2mks)
- (ii) Greens theorem. (2mks)

### QUESTION TWO (20 MARKS)

- (a) Find the dot product  $\mathbf{A} \cdot \mathbf{B}$  given that  $\mathbf{A} = 3\mathbf{i} + 3\mathbf{j}$  and  $\mathbf{B} = -5\mathbf{i}$ . (5mks)
- (b) Find  $\theta$  for non zero vectors  $\mathbf{A}$  and  $\mathbf{B}$  where  $\mathbf{A} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$  and  $\mathbf{B} = \mathbf{i} + \mathbf{j} + \mathbf{k}$ . (5mks)
- (c) Find a vector perpendicular to the plane determined by the 3 vectors  $\mathbf{A} = (1, 3, 2)$ ,  $\mathbf{B} = (4, -1, 1)$  and  $\mathbf{C} = (3, 0, 2)$ . (5mks)
- (d) Determine a unit vector perpendicular to the plane of  $\mathbf{P} = 2\mathbf{i} - 6\mathbf{j} - 3\mathbf{k}$  and  $\mathbf{Q} = 4\mathbf{i} + 3\mathbf{j} - \mathbf{k}$ . (5mks)

### QUESTION THREE (20 MARKS)

Use Green's Theorem to evaluate the following

- (a)  $\oint_c xydx - x^2y^3 dy$  where  $c$  is the triangle with vertices  $(0, 0)$ ,  $(0, 1)$  and  $(1, 1)$  positively oriented. (20mks)
- (b) Show how the Greens theorem may work with regions with holes. (20mks)

### QUESTION FOUR (20 MARKS)

- (a) State the divergence theorem.
- (b) Using four points describe the divergence theorem.
- (c) Use the divergence theorem to evaluate  $\iint_s \mathbf{F} \cdot d\mathbf{s}$  where  $\mathbf{F} = x\mathbf{i} - \frac{1}{2}y^2\mathbf{j} + z\mathbf{k}$  and the surface consists of 3 surfaces  $z = 4 - 3x^2 - 3y^2$ ,  $1 \leq z \leq 4$  on the top,  $x^2 + y^2 = 1$ ,  $0 \leq z \leq 1$  on the sides and  $z = 0$  on the bottom. (20mks)

### QUESTION FIVE (20 MARKS)

- (a) Find the gradient of the scalar field  $\mathbf{W} = 10r \sin^2\theta \cos\theta$ . (3mks)
- (b) Given  $\mathbf{Q} = x^2y^2 + xyz$ , compute  $\nabla \mathbf{W}$  and the direction derivative  $\frac{d\mathbf{W}}{dl}$  in the direction  $3\mathbf{a}_x + 4\mathbf{a}_y + 12\mathbf{a}_z$  at  $(2, -1, 0)$ . (3mks)
- (c) Find the angle at which the line  $x = y = 2z$  intersects the ellipsoid  $x^2 + y^2 + 2z^2 = 0$ . (14mks)