(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

SECOND YEAR FIRST SEMESTER

SPECIAL/SUPPLEMENTARY EXAMINATION

FOR THE DEGREES OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: MAT 201

COURSE TITLE: LINEAR ALGEBRA I

DATE: 17/10/2018 **TIME: 8 AM- 10 AM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

Question one (compulsory) **[30 marks]**

a) i) let V be a vector space. Given that the set $U = \{u_1, u_2, u_3, \ldots, u_n\}$ is a subset of V, explain the meaning of U is a basis of V. (3 mks)

ii) give the standard basis for R^2 and the standard basis for R^3. (5 mks)

b) state any three examples of a vector space. (3 mks)

c) determine whether the set of vectors in R^3 is linearly independent or linearly dependent $S = \{(1,2,3), (0,1,2), (-2,0,1)\}$ (9 mks)

d) define the following

i. nullity of a linear transformation $T: V \to W$ (3 mks)

ii. range of a linear transformation $T: V \to W$ (3 mks)

iii. rank of a linear transformation $T: V \to W$ (3 mks)

iv. dimension of a vector space. (2 mks)

Question Two (20 Marks)

a) the linear transformation $T: R^n \to R^m$ is defined by $T(v) = Av$. Find the dimensions of R^n and R^m for the linear transformation represented by each matrix

i. $A = \begin{pmatrix} 0 & 1 & -1 \\ 2 & 3 & 0 \\ 4 & 2 & 1 \end{pmatrix}$

ii. $A = \begin{pmatrix} 2 & -3 \\ -5 & 0 \\ 0 & -2 \end{pmatrix}$

iii. $A = \begin{pmatrix} 1 & 0 & -1 & 2 \\ 3 & 1 & 0 & 0 \end{pmatrix}$ (9 mks)

b) Show that the linear transformation $T: R^2 \to R^2$ represented by the matrix

$A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ has the property that it rotates every vector in R^2 counterclockwise about the origin through the angle θ. (11 mks)

Question three (20 marks)

a. When is an nxn matrix A invertible? (2 mks)

b. If A is an invertible matrix, then its inverse is unique. Prove (7 mks)

c. Show that B is the inverse of A where,

$$A = \begin{pmatrix} -1 & 2 \\ -1 & 1 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & -2 \\ 1 & -1 \end{pmatrix}$$ (5 mks)

d. Compute A^{-2} in two ways and show that the results are equal given that

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 4 \end{pmatrix}$$ (6 mks)

Question four (20 marks)

a) A non-empty subset u of a vector space V is a subspace of V iff

i) for any $x \in u$ and $y \in u$, the sum $x+y \in u$.

ii) for any $x \in u$, $\alpha x \in u$, α is a scalar. Prove (10 mks)

b) U, W are subspaces of a vector space V. Show that $U \cap W$ is also a subspace of V. (10 mks)

Question Five (20 marks)

a. Consider the set of vectors V = { [x,y,z]: ax+by+cz =0} where a, b, c are scalars. Show that V is a vector space. (10 mks)

b. Let V be a vector space, then

   i. $\alpha.0 = 0$ for every scalar α

   ii. $0.x = 0$ for every x in V

   iii. If $\alpha.x = 0$ then $\alpha = 0$ or $x = 0$

   (10 mks)