(*Knowledge for Development*)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2015/2016 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: MAT 201

COURSE TITLE: LINEAR ALGEBRA I

DATE: 10/5/16 **TIME:** 11.30AM -1.30PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of **4 Printed Pages**. Please Turn Over.

QUESTION ONE (30 MKS)

(a) (i) Define a vector space over $\mathbb{R}$. (3mks)

(ii) Suppose that the vector V is the set of positive real numbers $(i.e\ x > 0)$ with addition and scalar multiplication defined as follows,

$x + y = xy$ and $cx = x^c$ .Show that this set under this addition and scalar multiplication is a vector space. (6mks)

(b) Show that the function $h(x) = 4x^2 + 3x - 7$ lies in the space $span\{f, g\}$ generated by $f(x) = 2x^2 - 5$ and $g(x) = x + 1$. (3mks)

(c) (i) Distinguish between linear dependence and independence. (1mks)

(ii) Determine whether the set $\{(1,2,0), (0,1,-1), (1,1,2)\}$

is linearly dependent in $\mathbb{R}^3$. (2mks)

(d)Prove that $\{(1,3,-1), (2,1,0), (4,2,1)\}$ is a basis for $\mathbb{R}^3$. (3mks)

(e) (i)Show that the row space and the column space of a matrix A have the same dimension. (4mks)

(ii) Find the basis for the row space of the following matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 4 \\ 1 & 1 & 5 \end{pmatrix}$ and determine its rank. (3mks)

(f) (i) Define a linear transformation (1mks)

ii) Prove that the following transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ defined by $T(x, y) = (2x, x + y)$ is linear. (2mks)

(g) Define

i) Kernel

ii) Range (2mks)

QUESTION TWO (20 MKS)

(a) Find the kernel and the Range of the linear operator: $T(x, y, z) = (x, y, 0)$.(3mks)

(b) Determine the kernel and the Range of the transformation defined by the matrix $= \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & 1 \\ 1 & 1 & 4 \end{pmatrix}$. (6mks)

(c) Given $T: U \to V$ is a linear transformation , show that

$dimKer(T) + dimrange(T) = dimdomainT$. (7mks)

(d) Consider the linear transformation : $T(x, y, z) = (3x + 4y, 5x + 7y)$ of $\mathbb{R}^2 \to \mathbb{R}^2$.

(i) Prove that T is invertible and find the inverse of T. (3mks)

(ii) Determine the pre-image of the vector $(1,2)$. (1mks)

QUESTION THREE (20 MKS)

(a) i) Define a vector subspace (2mk)

ii) Show that the set $U$of a 2×2 diagonal matrices is a subspace of the vector space M_{22} of 2×2matrices .(3mks)

iii)Let W be a set of vectors of the form $(a, a, a + 2)$.

Show that W is not a subspace of $\mathbb{R}^3$.(3mks)

(b) i) Define a vector basis. (2mks)

ii) Let $\{v_1, , , , , , v_n\}$be a basis for a vector space V.If $\{w_1, , , , , w_n\}$ is a set of more than n vectors in V.Show that this set is linearly dependent. (3mks)

(c) Prove the following

i) the origin is a subspace of $\mathbb{R}^3$.

ii) the one dimensional subspaces of $\mathbb{R}^3$ are lines through the origin.

iii) the two dimensional subspaces of$\mathbb{R}^3$ are planes through the origin.(5mks)

(d) Let $\mathbb{R}^n \to \mathbb{R}^n$ be defined by $T(u) = A(u)$. Show that the range of T is spanned by the column vector of A. (3mks)

QUESTION FOUR (20MKS)

(a)Show that the linear transformation T is one-to-one if and only if the

kernel is a zero vector. (4mks)

(b) i) Define an invertible linear transformation on $\mathbb{R}^n$. (2mks)

ii) Let be a linear transformation of$\mathbb{R}^n \rightarrow \mathbb{R}^n$ .Show that T is invertible if and only if it is singular,its inverse is unique and it is linear. (7mks)

(c) i. Dfine a null space. (2mks)

ii. Show that a null space of an $m \times n$ matrix A is a subset of $\mathbb{R}^n$. (5mks)

QUESTION FIVE (20 MKS)

(a) Solve the following homogeneous system of linear equations.Interpret the set of solutions as a subspace and sketch the subspace of solutions.

$$x_1 + 2x_2 + 3x_3 = 0$$

$$-x_2 + x_3 = 0$$

$$x_1 + x_2 + 4x_3 = 0$$

(7mks)

(a) Analyse the solutions to the following system of equations

$$x_1 - 2x_2 + 3x_3 + x_4 = \ 1x_2 + 2x_3 - x_4 = 4$$

$$3x_1 - 5x_2 + 5x_3 = 5$$

$$x_1 - x_2 - 2x_4 = 3$$

(6mks)

(c) Consider the matrix $A = \begin{bmatrix} 1 & -1 & -2 \\ 2 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$.Compute the reduced echelon form of A. Discuss the implications of the answer. (4mks)

(d) . Find a matrix A such that $W = ColA$.

$$W = \left\{ \begin{bmatrix} 6a - b \\ a + b \\ -7a \end{bmatrix} : a, b \in \mathbb{R} \right\}$$

(3mks)

END