

207



(Knowledge for Development)

KIBABII UNIVERSITY
UNIVERSITY EXAMINATIONS
2017/2018 ACADEMIC YEAR
SECOND YEAR FIRST SEMESTER
MAIN EXAMINATION

**FOR THE DEGREE OF BACHELOR OF EDUCATION AND
BACHELOR OF SCIENCE**

COURSE CODE: MAT 201

COURSE TITLE: LINEAR ALGEBRA I

DATE: 15/01/18

TIME: 2 PM - 4 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 4 Printed Pages. Please Turn Over.

Question one (compulsory)**[30 marks]**

a) Let V be a vector space over a field F . given that $U = \{u_1, u_2, \dots, u_n\}$ is a subset of V , explain the meaning of the following; U is

- i. a subspace of V (2 mks)
- ii. linearly independent (2 mks)
- iii. a basis for V (2 mks)

b) consider the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 2 & 1 & 0 \end{pmatrix}$$

Find the reduced row echelon equivalence of A . (3 mks)

c) solve the system

$$x_2 + x_3 - 2x_4 = -3$$

$$x_1 + 2x_2 - x_3 = 2$$

$$2x_1 + 4x_2 + x_3 - 3x_4 = -2$$

$$x_1 - 4x_2 - 7x_3 - x_4 = -19 \quad (10 \text{ mks})$$

d) find the kernel of the linear transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ represented by

$$T(x_1, x_2) = (x_1 - 2x_2, 0, -x_1) \quad (5 \text{ mks})$$

e) Let $T: \mathbb{R}^5 \rightarrow \mathbb{R}^7$ be a linear transformation

- i. Find the dimension of the kernel of T if the dimension of the range is 2 (2 mks)
- ii. Find the rank of T if the nullity of T is 4 (2 mks)
- iii. Find the rank of T if $\ker(T) = \{0\}$ (2 mks).

Question two (20 marks)

- a) If W is a nonempty subset of a vector space V , then W is a subspace of V iff the following closure conditions hold.
- If u and v are in W , then $u+v$ is in W .
 - If u is in W and c is a scalar, then cu is in W . Prove (6 mks)
- b) Let W be the set of all 2×2 symmetric matrices. Show that W is a subspace of the vector space $M_{2,2}$, with the standard operations of matrix addition and scalar multiplication. (4 mks)
- c) If V and W are both subspaces of a vector space U , then the intersection of V and W is also a subspace of U . (6 mks)
- d) Show that the subset of \mathbb{R}^2 consisting of all points on the unit circle $x^2+y^2 = 1$ is not a subspace. (4 mks)

Question three (20 marks)

- a) Consider the set of vectors $V = \{[x, y, z] : ax + by + cz = 0\}$ where a, b, c are scalars. Show that V is a vector space. (10 mks)
- b) Let V be a vector space, then
- $\alpha \cdot 0 = 0$ for every scalar α
 - $0 \cdot x = 0$ for every x in V
 - If $\alpha \cdot x = 0$ then $\alpha = 0$ or $x = 0$ (10 mks)

Question four (20 marks)

- a) Show that the set of vectors

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \text{ spans } \mathbb{R}^3 \quad (5 \text{ mks})$$

- b) If A and B are invertible matrices of size n, then AB is invertible and $(AB)^{-1} = B^{-1}A^{-1}$

(6 mks)

- c) Determine whether the set of vectors in P_2 is linearly independent or linearly dependent $S = \{1+x-2x^2, 2+5x-x^2, x+x^2\}$

(9 mks)

Question Five (20 marks)

- a) When is an nxn matrix A invertible?

(2 mks)

- b) If A is an invertible matrix, then its inverse is unique. Prove

(7 mks)

- c) Show that B is the inverse of A where,

$$A = \begin{pmatrix} -1 & 2 \\ -1 & 1 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & -2 \\ 1 & -1 \end{pmatrix} \quad (5 \text{ mks})$$

- d) Compute A^{-2} in two ways and show that the results are equal given that

$$A = \begin{pmatrix} 1 & 1 \\ 2 & 4 \end{pmatrix} \quad (6 \text{ mks})$$