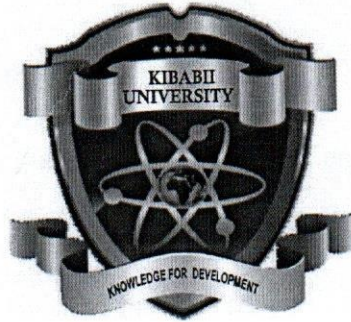


55



*(Knowledge for Development)*

**KIBABII UNIVERSITY**  
**UNIVERSITY EXAMINATIONS**  
**2017/2018 ACADEMIC YEAR**  
**SECOND YEAR SECOND SEMESTER**  
**MAIN EXAMINATION**

**FOR THE DEGREE OF BACHELOR OF EDUCATION AND  
BACHELOR OF SCIENCE**

**COURSE CODE: MAT 201**

**COURSE TITLE: LINEAR ALGEBRA I**

**DATE: 06/08/18**

**TIME: 2 PM -4 PM**

---

**INSTRUCTIONS TO CANDIDATES**

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

**QUESTION ONE (30 MARKS)**

- a. Give the definition of a vector space. (10mks)
- b. Show that the vector  $v_1$  is a linear combination of  $v_2$  and  $v_3$  given that  $v_1=(1,3,1)$ ,  $v_2=(0,1,2)$  and  $v_3=(1,0,-5)$ . (4mks)
- c. Find the null space of the matrix  $A = \begin{bmatrix} 1 & 2 & -2 \\ 3 & 6 & -5 \\ 1 & 2 & 0 \end{bmatrix}$ . (4mks)
- d. Find the coordinate matrix  $P=3x^3-2x^2+4$  relative to the standard basis in  $P_3$ ,  $S=\{1,x,x^2,x^3\}$ . (4mks)
- e. Define the basis of a vector space. (3mks)
- f. Show that the following set is a basis for  $\mathbf{R}^3$ .  $S=\{(1,0,0),(0,1,0),(0,0,1)\}$ . (5mks)

**QUESTION TWO (20 MARKS)**

Show that a set of a  $2 \times 3$  matrices with operations of matrix addition and scalar multiplication is a vector space. (20mks)

**QUESTION THREE (20 MARKS)**

- a. Define a linear transformation. (3mks)
- b. A linear transformation is defined by a matrix function  $T: \mathbf{R}^2 \rightarrow \mathbf{R}^3$  is defined as  $T(v) = Av = \begin{pmatrix} 3 & 0 \\ 2 & 1 \\ -1 & -2 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$
- (i) Find  $T(v)$ , where  $v=(2,-1)$  (3mks)
- (ii) Show that  $T$  is a linear transformation from  $\mathbf{R}^2$  into  $\mathbf{R}^3$ . (3mks)
- c. Find the kernel of the linear transformation  $T: \mathbf{R}^3 \rightarrow \mathbf{R}^2$  defined by  $T(x) = Ax$  where  $A = \begin{pmatrix} 1 & -2 & -2 \\ -1 & 2 & 3 \end{pmatrix}$  (11mks)

**QUESTION FOUR (20 MARKS)**

- a. Determine if the following vectors from  $\mathbf{R}^3$  are linearly independent linearly dependent. (10mks)
- b. Find the rank and nullity of the matrix  $B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 5 \\ 0 & 1 & 3 \end{bmatrix}$  (10mks)

**QUESTION FIVE (20 MARKS)**

- a. Let  $D$  be a linear operator from  $P_3$  to  $P_2$  defined by  $D(a_0 + a_1x + a_2x^2 + a_3x^3) = a_1 + 2a_2x + 3a_3x^2$ . Find the matrix representative of  $D$  with respect to the basis  $B = \{1, x, x^2, x^3\}$  of  $P_3$  and the basis  $B^1 = \{1, x, x^2\}$  of  $P_2$ . Use matrix to find  $D(5 - 7x + 11x^2 + 4x^3)$ . **(10mks)**
- b. Let  $B = \{(-3, 2), (4, 2)\}$  and  $B^1 = \{(-1, 2), (2, -2)\}$  be bases for  $\mathbf{R}^2$  and let  $A = \begin{pmatrix} -1 & 4 \\ 5 & -3 \end{pmatrix}$  be the matrix for  $T: \mathbf{R}^2 \rightarrow \mathbf{R}^2$  relative to  $B$ . Find the matrix relative to  $B^1$ . **(10mks)**