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(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: **MAT 201**

COURSE TITLE: **LINEAR ALGEBRA I**

DATE: 06/08/18 **TIME: 2 PM -4 PM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

a. Give the definition of a vector space. **(10mks)**

b. Show that the vector $\mathbf{v_1}$ is a linear combination of $\mathbf{v_2}$ and$\mathbf{v_3}$ given that $\mathbf{v_1}=(1,3,1)$, $\mathbf{v_2}=(0,1,2)$ and $\mathbf{v_3}=(1,0,-5)$. **(4mks)**

c. Find the null space of the matrix A= $\begin{bmatrix} 1 & 2 & -2 \\ 3 & 6 & -5 \\ 1 & 2 & 0 \end{bmatrix}$. **(4mks)**

d. Find the coordinate matrix $P=3x^3-2x^2+4$ relative to the standard basis in P_3, $S=\{1,x,x^2,x^3\}$.**(4mks)**

e. Define the basis of a vector space. **(3mks)**

f. Show that the following set is a basis for $\mathbf{R}^3$. S={(I,0,0),(0,1,0),(0,0,1)}. **(5mks)**

QUESTION TWO (20 MARKS)

Show that a set of a 2×3 matrices with operations of matrix addition and scalar multiplication is a vector space. **(20mks)**

QUESTION THREE (20 MARKS)

a. Define a linear transformation. **(3mks)**

b. A linear transformation is defined by a matrix function $T:\mathbf{R}^2\rightarrow\mathbf{R}^3$ is defined as T(v)= Av= $\begin{pmatrix} 3 & 0 \\ 2 & 1 \\ -1 & -2 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$

(i) Find T(v), where v=(2,-1) **(3mks)**

(ii) Show that T is a linear transformation from $\mathbf{R}^2$ into $\mathbf{R}^3$. **(3mks)**

c. Find the kernel of the linear transformation $T:\mathbf{R}^3\rightarrow\mathbf{R}^2$ defined by T(x)= Ax where A= $\begin{pmatrix} 1 & -2 & -2 \\ -1 & 2 & 3 \end{pmatrix}$ **(11mks)**

QUESTION FOUR (20 MARKS)

a. Determine if the following vectors from $\mathbf{R}^3$ are linearly independent linearly dependent. **(10mks)**

b. Find the rank and nullity of the matrix B= $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 5 \\ 0 & 1 & 3 \end{bmatrix}$ **(10mks)**

QUESTION FIVE (20 MARKS)

a. Let D be a linear operator from P_3 to P_2 defined by $D(a_0 + a_1x + a_2x^2 + a_3x^3) = a_1 + 2a_2x + 3a_3x^2$. Find the matrix representative of D with respect to the basis $B = \{1, x, x^2, x^3\}$ of P_3 and the basis $B^1 = \{1, x, x^2\}$ of P_2. Use matrix to find $D(5 - 7x + 11x^2 + 4x^3)$. **(10mks)**

b. Let $B = \{(-3,2),(4,2)\}$ and $B^1 = \{(-1,2),(2,-2)\}$ be bases for $\mathbf{R}^2$ and let $A = \begin{pmatrix} -1 & 4 \\ 5 & -3 \end{pmatrix}$ be the matrix for $T: \mathbf{R}^2 \rightarrow \mathbf{R}^2$ relative to B. Findthe matrix relative to B^1. **(10mks)**