(*Knowledge for Development*)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

FIRST YEAR SECOND SEMESTER

SPECIAL/SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: **MAT 123**

COURSE TITLE: LINEAR ALGEBRA

DATE: 17/10/18 **TIME:** 8 AM -10 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

a) For which rationals a and b does the following system have
   i) No solution
   ii) A unique solution
   iii) Infinitely many solutions

$$x_1 - 2x_2 + 3x_3 = 4$$
$$2x_1 - 3x_2 + ax_3 = 5$$
$$3x_1 - 4x_2 + 5x_3 = b$$

(5 marks)

b) Find the inverse of the matrix A by first getting the adjoint.

$$A = \begin{pmatrix} 1 & 3 & -4 \\ 0 & -2 & 3 \\ 1 & 1 & 1 \end{pmatrix}$$

(6 marks)

c) Distinguish between linear dependence and linear independence of vectors in a vector space and hence determine if the set $S = \{2 + x + x^2, x - 2x^2, 2 + 3x - x^2\}$ is linearly independent in P_2 (polynomials of second degree) (5 marks)

d) Solve using Cramer's rule

$$2x_1 + x_2 - 2x_3 = 10$$
$$3x_1 + 2x_2 + 2x_3 = 1$$
$$5x_1 + 4x_2 + 3x_3 = 4$$

(6 marks)

e) Let the vector space V be the set of all polynomials of degree 3 and W be the set of all polynomial of degree 3 or less but with a constant zero term. Is W a subspace of V. (3 marks)

f) Define the term linear transformation and hence determine if $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined as $T(x_1, x_2) = (x_1 + x_2, 2x_1 - x_2)$ is a linear transformation. (5 marks)

QUESTION TWO (20 MARKS)

a) i) Find the rational number t for which the following system is consistent and solve the system for this value of t

$$x_1 + x_2 = 2$$
$$x_1 - x_2 = 0$$
$$3x_1 - x_2 = t$$

ii) Reduce to echelon form and hence find the rank of the matrix

$$A = \begin{pmatrix} 3 & 1 & 2 \\ 2 & 1 & 1 \\ 4 & 2 & 2 \end{pmatrix}$$ (6 marks)

b) If A_{nxn} is a matrix, show that the following statements are equivalent.

i) A is invertible

ii) $A\underline{X} = \underline{b}$ has a unique solution for any $\underline{b}$.

iii) $A\underline{X} = \underline{O}$; has a trivial solution only.

iv) A is row equivalent to I_n. (7 marks)

c) Solve by getting the inverse

$$x_1 - 2x_2 + x_3 = 7$$
$$2x_1 - x_2 + 4x_3 = 17$$
$$3x_1 - 2x_2 + 2x_3 = 17$$
(6 marks)

QUESTION THREE (20 MARKS)

a) i) If a matrix A_{nxn} is invertible, then show that the inverse is unique. (3 marks)

ii) Find the inverse of the matrix below by first appending an identity matrix I to the right hand side of A and reducing the left hand side of $\left[A|I\right]$ to identity matrix.

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 3 & 2 \\ 1 & 0 & 3 \end{pmatrix}$$ (4 marks)

b) Let the vector space $V = \mathbb{R}^4$ and $S = \{(1,-2,0,3), (2,3,0,-1), (2,-1,2,1)\}$. Determine if $(3,9,-4,-2) \in L(S)$ (where $L(S)$ is the set spanned by S). (4 marks)

c) Determine whether or not the following vectors in $\mathbb{R}^3$ are linearly dependent
$u_1 = (1,-2,1), u_2 = (2,1,-1), u_3 = (7,-4,1)$ (4 marks)

d) If $S = \{v_1, v_2, ..., v_n\}$ is a basis for a vector space V then show that every set with more than n vectors is linearly dependent. (5 marks)

QUESTION FOUR (20 MARKS)

a) Find the basis and dimension of the solution space for the equations

$$2x_1 + 2x_2 - x_3 + x_5 = 0$$
$$-x_1 - x_2 + 2x_3 - 3x_4 + x_5 = 0$$
$$x_1 + x_2 - 2x_3 - x_5 = 0$$
$$x_3 + x_4 + x_5 = 0$$

(5 marks)

b) For the vectors $v_1 = (1,1,-1), v_2 = (4,0,1), v_3 = (3,-1,2)$

i) Find a basis for the subspace spanned by these vectors.

ii) Write the remaining vectors as a linear combination as the vectors in the basis.

(4 marks)

c) Given that $T : \mathbb{R}^3 \to \mathbb{R}^3$ is defined as

$T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2x_1 + x_2 - x_3 \\ -x_3 + x_2 \\ x_1 - x_2 - x_3 \end{pmatrix}$, find the matrix of T with respect to standard basis.

(3 marks)

d) Given $v_1 = t^3 - 2t^2 + 4t + 1;\ v_2 = 2t^3 - 3t^2 + 9t - 1;\ v_3 = t^3 + 6t - 5;\ v_4 = 2t^3 - 5t^2 + 7t + 5$

iii) Find a basis for the subspace spanned by these vectors.

iv) Write the remaining vectors as a linear combination of the vectors in the basis.

(6 marks)

e) Given that $T : \mathbb{R}^3 \to \mathbb{R}^3$ is defined as $T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2x - 3y + 4z \\ 5x - y + 2z \\ 4x + 7y \end{pmatrix}$, find the matrix of T with respect to standard basis.

(2 marks)

QUESTION FIVE (20 MARKS)

a) By first getting the adjoint, find the inverse of the matrix

$A = \begin{pmatrix} 1 & -2 & 1 \\ 2 & -1 & 4 \\ 3 & -2 & 2 \end{pmatrix}$ and hence solve by inverse method

$$x - 2y + z = 7$$
$$2x - y + 4z = 17$$
$$3x - 2y + 2z = 14$$

(9 marks)

b) Let $T:\mathbb{R}^3 \to \mathbb{R}^3$ be defined as

$$T(X) = \begin{pmatrix} 2 & 0 & -1 \\ 4 & 0 & -2 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}. \text{ Find}$$

i) Basis for image of T

ii) Basis for Kernel of T

v) Rank and nullity of T.

(7 marks)

c) Define the following

i) Symmetric Matrix

ii) Homogenous system

iii) Transpose of a matrix

iv) Elementary matrices

(4 marks)