(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2016/2017 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: MAT 213

COURSE TITLE: LINEAR ALGEBRA II

DATE: 20/09/17 **TIME:** 11.30 AM -1.30 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of **4 Printed Pages**. Please Turn Over.

QUESTION 1: COMPULSORY (30 MKS)

a) Determine the standard matrix for a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ defined by

$T(x, y, z) = (x - 2y, 2x + y)$ **(5 mks)**

b) If u and v are non – zero orthogonal vectors in an inner product space V then prove that $||u + v||^2 = ||u||^2 + ||v||^2$ **(6 mks)**

c) Show that the function defined by $< u, v > = u_1v_1 + 2u_2v_2$ is an inner product on $\mathbb{R}^2$ **(6 mks)**

d) Let u and v be vectors in an inner product space V. If $u = (1, -1, 2)$ and $v = (0, 12, 0)$. Find

i) $||u||$ **(3 mks)**

ii) $||v||$ **(3 mks)**

iii) $d(u, v)$ **(3 mks)**

iv) The angle between u and v **(4 mks)**

QUESTION 2 : (20 MKS)

a). Find the orthogonal projection of u *and* v in $\mathbb{R}$, given that

$u = (4, 2)$ and $v = (3, 4)$ **(4 mks)**

b) Let w be a subspace of $\mathbb{R}^5$ spanned by $v_1 = (1, 2, 3, -1, 2)$ $v_2 = (2, 4, 7, 2, -1)$.
Find a basis of the orthogonal complement $w^{\perp}$ of w

(8 mks)

c) Define the terms : linear transformation ,orthogonal vectors and norm of a vector **(8 mks)**

QUESTION 3 : (20 mks)

a) Prove that the set $S = \{v_{1,} v_{2,} v_{3,} v_4\} \in \mathbb{R}^4$ is a basis for $\mathbb{R}^4$ where

$v_1 = (2,3,2,-2)$, $v_2 = (1,0,0,1)$, $v_3 = (-1,0,2,1)$ and
$v_4 = (-1,2,-1,1)$

(10 mks)

b) Apply Gram-Smchimdt orthonormalization process on a basis
$B = \{(1,1,1\ ,\ 1)\ ,\ (1,2,4\ ,\ 5)\ ,\ (1,-31,-4\ ,-2)\}$ for $\mathbb{R}^3$ to find the Orthonormal basis of the set B. **(10 mks)**

QUESTION 4 : (20 MKS)

Given the square matrix $A = \begin{bmatrix} 1 & -3 & 3 \\ 0 & -5 & 6 \\ 0 & -3 & 4 \end{bmatrix}$ Find the matrix P such that $B = P^{-1}AP$ where P is a diagonal matrix . Hence determine the matrix B. **(20 mks)**

QUESTION 5 : (20 mks)

a) Define the term inner product space **(4 mks)**

b) Find the eigenvectors corresponding to eigenvalues of the matrix $A = \begin{bmatrix} 2 & 3 & -2 \\ 1 & 4 & -2 \\ 2 & 10 & -5 \end{bmatrix}$

(16 mks)