(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: **MAT 213**

COURSE TITLE: LINEAR ALGEBRA II

DATE: 10/10/18 **TIME:** 11.30 AM -1.30 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of **4 Printed Pages**. Please Turn Over.

QUESTION ONE: COMPULSORY (30 MARKS)

(a)Using illustration define the norm of a vector X denoted by $||X||$ (2mks)

(b)Let x , y be ∈ C . Define a dot product as X.Y=$||X||$ $||Y||$. State the characteristics of the dot product over the complex plane.(4mks)

(c)If e_1---------e_m is an orthonormal list of vectors in V then for a_1,......................a_m∈ R ,show that

$||a_1 \, e_1 + - - - - - - - - + \, a_m \, e_m||^2$=$||a||,^2$ + ------+$||a_m||^2$ (2mks)

(d)Show that $(\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2})$ and $(\frac{1}{2} \; \frac{1}{2} \; \frac{-1}{2} \; \frac{-1}{2})$ is an orthonomal basis in F^4.(4mks)

(e)Define the eigen value and eigen vector of a square matrix (nxn) A.(4 mks)

(d)Show that vector O is orthogonal to every vector . (4mks)

(c)Let X,Y and Z ∈ V where V is a vector space over F.

Show that <X,Y+Z>=<X,Y>+<X,Z>. (5mks)

(b) Suppose e_1---------e_n is orthornomal basis of V and v ∈ V , Write V as a linear combination of e_i and state its norm (5mks)

QUESTION TWO(20 mks)

Given a system of linear equations below.

(a) $-2x_1 + x_2 - x_3 = -4$

$x_1 + 2x_2 + 3x_3 = 13$

$3x_1 + x_3 = -1$

Use Gausian elimination to solve the systems of equations above .(10mks)

(b)Using pythagoras theorem show that $||u + v||^2 = ||u||^2 + ||v||^2$ where u and v are orthogonal vectors in the vector space V .(6mks) (c)Let V be a vector space defined over k. Define the inner product of V.(4mks)

QUESTION THREE (20 marks)

(a) State and prove the five properties of an inner product space. (10 mks)

(b) Let A be defined by $\begin{bmatrix} 1 & 4 \\ 3 & 5 \end{bmatrix}$

Calculate the eigenvectors and eigenvalues of A.(10MKS)

QUESTION FOUR (20 marks)

(a) Show that the following lists of vectors are orthonormal (10mks)

$\{e_1=(1,0,0)\ e_2=\{0,1,0\}\ e_3=(0,0,1).\}$

$\{(\frac{1}{-\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})\ (\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0)\ (\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}})\}.$

(b) Let f be a linear functional on a normed space V . Show that the following are equivalent.

I. f is continous. (2mks)
II. f is continous at 0 (3mks)

(c) Giving examples define a linear functional f over the vector space V .(5mks)

QUESTION FIVE (20 marks)

(a) Let $T:U \longrightarrow V$ be a map from vector space U to vector space V . State two conditions that must be met by T for it to be a linear transformation. (2mks)

(b)Define T:R3 $\longrightarrow$ R3 by describing the output of the function with the formular

$$T\left(\begin{vmatrix} X_1 \\ X_2 \\ X_3 \end{vmatrix}\right) = \begin{bmatrix} 2X_1 & + & X_3 \\ & -4X^2 & \end{bmatrix}$$

Show that T is a linear transformation.(12mks)

(c) Define the norm of a linear functional f in a normed space V .(3mks)

(d)Let A = (2X2) be a matrix with eigenvalues and eigenvectors. Define a characteristic equation for the matrix. (3mks)