(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: MAT 213

COURSE TITLE: LINEAR ALGEBRA II

DATE: 30/07/18 **TIME:** 9 AM - 11 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE COMPULSORY (30 MARKS)

(a) Letf(x)=x be a function over the vector space V of continuous real valued functions.

Define the inner product of the function f(x)over the interval (-1,1) as $< f.f >= \int_{-1}^{1}(fx\ )2$ dx

Show that$< f.f >$is an inner product. (4mks)

(b) Let X,Y $\in R$, .Define ,a dot product of ,X and Y denoted by X.Y and give its four properties. (5mks)

(c)Let X,Y and Z $\in$ V where V is a vector space over F.

Show that$< X, X + Y > = < X, X > + < X, Y >$ (4 mks)

(d)Show that vector $\bar{O}$ is orthogonal to every vector . (3mks)

(e) Let $e_1.........e_m$ be an orthonormal list of vectors in V.

Show that$||ae_1 + \ldots\ldots\ldots.. ae_n||^{2=} ||a||^2+..........+ ||a||^2$ (2mks

(f)Suppose A is a square matrix (nxn) . Define eigenvector and eigenvalue over the matrix A.

(g)Define the norm of a linear functional f in a normed space V. (3MKS)

(h) Given a system of linear equations below.

$-2x_1 + x_2 + x_3 -4$

$X_1 + 2x_2 + 3x_{3\ =13}$

$3x_1 + x_3 = -1$

Use Gausian elimination to solve the systems of equations above . (10mks)

(i)Using pythagoras theorem show that II u+vII^2= =IIu II ^{2} + II vII^2 where u and v are orithogonal vectors in the vector space V . (5mks)

QUESTION TWO (20 MARKS)

(a).Let (V <.>)be an inner product space over R.Define a function $||V|| = \sqrt{< V.V >}$

Show that$||.||$ is a norm on V . (16mks)

(b) Let V be a vector space over the field F^n. Let e_1----$e_m$be orthonormalbasis of V and v $\in$V. Express the vector v in terms of the orthonormal basis e_1-----e_m and state its norm.

(4mks)

QUESTION THREE (20 MARKS)

(a) Define orthonormal vectors (2mks)

(b)Show that the following lists of vectors are orthonormal (12mks)

{e_1=(1,0,0) e_2={0,1,0} e_3=(0,0,1).

$$\left\{\left((\frac{1}{-\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right)(\frac{-1}{\sqrt{2}},\frac{1}{\sqrt{2}}\ ,0)\ (\frac{1}{\sqrt{6}},\frac{1}{\sqrt{6}},\frac{-2}{\sqrt{6}})\right\}$$

(c) Define the normed dual of a normed vector space V giving an illustration (4mks)

(d)Prove that everyorthonormal list of vectors is linearly independent (3mks)

QUESTION FOUR (20 MARKS)

(a) Let T:$U \rightarrow$V be a map from vector space U to vector space V . State two conditions that must be met by T for it to be a linear transformation. (2mks)

(b)Define T :$R^3 \rightarrow R^2$ by describing the output of the function with the formular

$$T\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} \rightarrow \begin{bmatrix} 2X_1 & + & X_3 \\ & -4X_2 & \end{bmatrix}$$

Show that T is a linear transformation. (12mks)

(c) Illustrate three examples of linear functionals (6mks)

QUESTION FIVE (20 MARKS)

(a)Let A = (2X2) be a matrix with eigenvalues and eigenvectors. Define a characteristic equation for the matrix. (4mks)

(b)Let A =$\begin{bmatrix} 1 & 4 \\ 3 & 5 \end{bmatrix}$ be the matrix defined above .

Calculate the eigenvectors and eigenvalues of A. (16mks)