(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

FIRST YEAR SECOND SEMESTER

SUPPLEMENTARY/SPECIAL EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND

BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: MAT 122

TITLE: ELEMENTARY APPLIED MATHEMATICS

DATE: 18/10/2018   **TIME:** 8 AM- 10 AM

INSTRUCTIONS TO CANDIDATES

Answer Question **One** and **Any other TWO** Questions

TIME: 2 Hours

QUESTION 1 (30 MARKS)

a) Find the angle between planes $3x + 2y - 2z = 4$ and $2x + y + 2z = 1$ (3marks)

b) Find the centre and radius of the circle passing through the points $(4,3)$, $(0,1)$ and $(1,0)$. (6 marks)

c) Determine if the line with parametric equations $x = -2t$, $y = 2 + 7t$, $z = -1 - 4t$ intersects the plane $4x + 9y - 2z = -8$. If it does give the coordinates of that point of intersection. (4 marks)

d) Find Cartesian equation for the curve $r = 2\sin\theta + 2\cos\theta$ (3marks)

e) For what values of x are the vectors $\langle -6, x, 2 \rangle$ and $\langle x, x^2, x \rangle$ orthogonal? (4 marks)

f) Given $\mathbf{a} = 3\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$, $\mathbf{b} = -\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and $\mathbf{c} = 3\mathbf{i} - \mathbf{k}$, find the angle between $\mathbf{a} + \mathbf{b}$ and $\mathbf{b} - 2\mathbf{c}$. (6 marks)

g) A ball of mass $72\,g$, travelling horizontally at $39\,ms^{-1}$ strikes a wall at right angles and rebounds with a speed of $28\,ms^{-1}$. Find the impulse exerted on the ball. (4marks)

QUESTION 2 (20 MARKS)

a) Derive the equation of motion $S = ut + \dfrac{1}{2}at^2$. (4 marks)

b) A stone is projected vertically upwards with a velocity of $90m/s$ from the ground. Calculate:

    i.   The time it takes to reach the maximum height . (2 marks)

    ii.   The time of flight. (3 marks)

    iii.   The maximum height reached. (3 marks)

    iv.   The velocity with which it lands on the ground. (3 marks)

c) Show from Newton's second law of motion that Force = mass × acceleration. (5 marks)

QUESTION 3 (20 MARKS)

a) Write down the equation of the plane containing the points $P(4, -3, 1)$, $Q(-3, -1, 1)$ and $R(4, -2, 8)$.

(5 marks)

b) Find the distance between the parallel planes $10x + 2y - 2z = 10$ and $5x + y - z = 4$

(5 marks)

c) i) Show that $a \times a = 0$ for any vector a.

(4 marks)

ii) Prove that the vector $\mathbf{a} \times \mathbf{b}$ is orthogonal to vector $\mathbf{b}$.

(6 marks)

QUESTION 4 (20 MARKS)

a) Is the point $(1,-1)$ within or outside the circle whose equation is $4x^2 + 4y^2 + 12x - 20y + 25 = 0$ (5 marks)

b) Show that the equation of a circle that passes through $(-1, 1)$ and whose centre is the point of intersection of the lines $x + 3y + 7 = 0$ and $2y - 3x + 12 = 0$ is given by $x^2 + y^2 - 4x + 6y - 12 = 0$

(6 marks)

(c) (i) Show that a particle has the same velocity at a height of S metres above the ground when it is ascending and when it is descending.

(5 mks)

(ii) Show that if a body leaves with a velocity of 10 m/s from the ground, it will arrive at the same spot with the same velocity downwards.

(4 mks)

QUESTION 5 (20 MARKS)

a) Represent the point with Cartesian coordinates $(1, -1)$ in terms of polar coordinates

(4 marks)

b) Convert $r = -7\cos\theta$ into Cartesian equation.

(4 marks)

c) Find the area enclosed by one loop of the four-leaved rose $r = \sin 2\theta$

(6 marks)

d) Calculate the arc length of the polar curve $r = -4\sin\theta$, $0 \leq \theta \leq \pi$

(6 marks)