(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2016/2017 ACADEMIC YEAR

SECONDYEAR FIRST SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: **MAT 212**

COURSE TITLE: LINEAR ALGEBRA

DATE: 18/09/17 **TIME:** 3 PM -5 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MKS)

(a) (i)Define subspace of vector spaces and a basis of a vector space (2mks)

(ii) Show that $V_4 \equiv$ the set consisting of all real valued continuous functions defined on the entire real line is a vector space. (5mks)

(b) Let W= {$(x_1, x_1 + x_3, x_3)$: x_1 and x_3 are real numbers}. Show that W is a subspace of R^3. (4mks)

(c) Solve the system below using Gaussian Elimination with back-substitution. (4mks)

x-2y+3z = 9

-x+3y =-4

2x-5y+5z=17

(d) (i) Show that the set S = {(1,1), (1,-1)} is a basis for R^2 (4mks)

(ii) Determine whether the set of vectors below is linearly independent or dependent (5mks)

$$S = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix}$$

(e) (i) Find the kernel of the linear transformation T: $R^2 \rightarrow R^3$ represented by T $(x_1, x_2) = (x_1 - 2x_2, 0, -x_1)$. (3mks)

(ii) Show that the linear transformation T: $R^2 \rightarrow R^2$ represented by the matrix

$$A = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$$

Has the property that it rotates every vector in R^2 counter clockwise about the origin through angle θ (3 mks)

QUESTION TWO (30 MKS)

(a) Use Gauss – Jordan elimination to solve the system

$$\begin{aligned} x+2y-2z &= -3 \\ w + 2x - y &= 2 \\ 2w + 4x + y - 3z &= -2 \\ w - 4x - 7y - z &= -19 \end{aligned}$$

(6mks)

(b) Write the vector w = (1, 1, 1) as a linear combination of vectors in the set S, where

S = {(1, 2, 3), (0, 1, 2), (-1, 0, 1)} (7 mks)

2(c) Proof that S= v_1, ………,v_n form a basis of vector space V if and only if every v ∈ Vcan be written uniquely as v =α_1 v_1 + α_2 v_2 +…………..+α_n v_n : the coefficients α_1, α_2, …………..,α_n are uniquely determine by the vector v. (4mks)

2(d) Proof that if W_1 and W_2 are subspaces of V then so is $W_1 \cap W_2$. (3 mks)

QUESTION THREE (30 MKS)

(a) Let U and W be subspace of a vector space V. Show that:

(i) U + V is a subspace of V (3mks)

(ii) U and W are contained in U + W (2mks)

(iii) U + W is the smallest subspace containing U and W; that is U + W = span (U, W). (2mks)

(iv) W + W = W (2 mks)

(b) Solve the system of linear equation below

$$\begin{aligned} x - y + 3z &= 0 \\ 2x + y + 3z &= 0 \end{aligned}$$

(5mks)

(c) Proof that:

(i) If A is an invertible matrix, then its inverse is unique. (3mks)

(ii) If A, B and C are invertible matrix and AC = BC , then A = B (3mks)

QUESTION FOUR (30 MKS)

(a) Determine whether the set of vectors S in R^3 is linearly independent or linearly dependent. Where, S = {(1,2,3), (0,1,2), (-2,0,1)} (6 mks)

(b) Find the inverse of the matrix below (6 mks)

$$A = \begin{pmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ -6 & 2 & 3 \end{pmatrix}$$

(c) Proof that if S = $\{v_1, v_2, \ldots\ldots\ldots, v_k\}$ is a set of vectors in a vector space V, then span(S) is a subspace of V. (4 mks)

(d) Proof that a set S = $\{v_1, v_2, \ldots, v_n\}$, n≥ 2, is linearly dependent if and only if at least one of the vectors v_j can be written as a linear combination of the other vectors in S. (4mks)

QUESTION FIVE (30 MKS)

(a) Solve the system (7mks)

u - v + 2w = 4

u + w = 6

2u + 3v + 5w = 4

3u + 2v − w = 1

(b) Proof that if the vectors $v_1, v_2, \ldots, v_n$, w span V and that w is a linear combination of $v_1, v_2, \ldots, v_n$, then $v_1, v_2, \ldots, v_n$, span V. (6mks)

(c) Find the kernel of the linear transformation, T: $R^3 \rightarrow R^2$ defined by T(x) = A(x), where

$$A = \begin{matrix} 1 & -1 & -2 \\ -1 & 2 & 3 \end{matrix}$$

(7 mks)