(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

SECONDYEAR FIRST SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: MAT 212

COURSE TITLE: LINEAR ALGEBRA

DATE: 08/10/18 **TIME:** 3 PM -5 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

(a) Show that P_2 is a vector space where P_2 is defined as a set of polynomials of the form $P(x) = a_2 x^2 + a_1 x + a_0$ where a_2, a_1 and a_0 are real numbers.

(6 mks)

b) Determine whether $T : \mathbb{R}^2 \to \mathbb{R}^3$ defined by $T(x_1 \quad x_2) = [x_2, \quad x_1 - \quad x_2, \quad 2x_1 + \quad x_2]$ is a linear transformation. (5mks)

c) Determine whether the subspace W of $\mathbb{R}^3$ of vectors of the form $\vec{u} = (u, 3u, 6u)$ is a subspace of $\mathbb{R}^3$ (6mks)

d) Solve the system below using Gaussian Elimination with back-substitution. (7mks)

$$x - 2y + 3z = 9$$
$$-x + 3y = -4$$
$$2x - 5y + 5z = 17$$

e) Let v be an element in a vector space V and let k be any scalar. Then prove that $c0 = 0$ where 0 is a zero vector. (6 mks)

QUESTION TWO (20MARKS)

a) Use Gauss – Jordan elimination to solve the system

$$a - 3c = -5$$
$$2a + 3b - c = 7$$
$$4a + 5b - 2c = 10$$

(10mks)

b) If A is a matrix given by $B = \begin{bmatrix} 1 & 5 & -2 & 3 & 5 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ write down a basis for the row space, a basis for the column space and a basis for the null space of the matrix B above.

(6mks)

c) Find the inverse of the matrix below using row reduction method $P = \begin{bmatrix} 1 & 3 & 1 \\ 1 & 0 & 4 \\ 3 & 4 & -1 \end{bmatrix}$

(4 mks)

QUESTION THREE (20 MARKS)

(a) Given the matrix $B = \begin{bmatrix} 1 & -2 & 1 & -5 & 6 \\ 4 & -4 & -4 & -12 & -8 \\ 2 & 0 & -6 & -2 & 4 \\ -3 & 1 & 7 & -2 & 12 \end{bmatrix}$ Find

i) a basis for the row space

ii) a basis for the column space

iii) a basis for the null space

iv) Hence find the rank and nullity of B (10 mks)

(b) Determine whether the two subsets are subspaces of $\mathbb{R}^2$ or not.

i) The set of all points on the line $5x + 6y = 0$ (6mks)

ii) The set of points on the line $2x - 5y = 47$

(4mks)

QUESTION FOUR (20 MARKS)

a) Compute $4C - 2D$ given that $C = \begin{bmatrix} 2 & -1 \\ 0 & 6 \\ -3 & 2 \end{bmatrix}$ and $D = \begin{bmatrix} -4 & 2 \\ 3 & 5 \\ -1 & -3 \end{bmatrix}$ (3mks)

b) Show whether the span of the vectors $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 3 \\ -5 \\ 4 \end{bmatrix}$ *and* $\begin{bmatrix} 4 \\ -3 \\ 9 \end{bmatrix}$ is all of $\mathbb{R}^3$ (5mks)

c) (i) Find the kernel of the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ represented by $T(x_1, x_2) = (x_1 - 2x_2, 0, -x_1)$. (3mks)

(ii) Determine whether the vector $[1, \ -7, \ -4]$ is in the span of vectors $[2, \ 1, \ 1]$ and $[1, \ 3, \ 2]$ (4mks)

d) Determine whether the set of vectors $\{[2, \ 1, \ -3], [4, \ 0, \ 2], [2, \ -1, \ 3]\}$ form a basis for the subspace of $\mathbb{R}^3$ that the vectors span (5mks)

QUESTION FIVE (20 MARKS)

a) Let $v = (v_1 , v_2)$ and $w = (w_1 , w_2)$ be any vectors in $\mathbb{R}^2$ and let k and m be any two scalars in $\mathbb{R}$. Prove the following properties

i) $v + w = w + v$

ii) $(k + m)w = kw + mw$ (6mkS)

b) Find the standard matrix representation of the linear transformation $T : \mathbb{R}^4 \rightarrow \mathbb{R}^3$ where $T([x_1 , x_2 , x_3 , x_4]) = [x_2 - 3x_3 , 2x_1 - x_2 + 3x_4 , 8x_1 - 4x_2 + 3x_3 - x_4]$ (4mks)

c) Find the values of a, b , c, d ,e and f elements in the 4x4 matrix below so that the matrix is symmetric

$$\begin{bmatrix} 5 & -6 & a & 8 \\ b & 3 & c & d \\ -2 & 1 & 0 & 4 \\ e & 11 & f & -1 \end{bmatrix}$$

(4mks)

d) Write down the given matrix below in echelon form.

$$\begin{bmatrix} 2 & -4 & 2 & -2 \\ 2 & -4 & 3 & -4 \\ 4 & -8 & 3 & -2 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

(6mks)