(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

SECOND YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: MAT 212

COURSE TITLE: LINEAR ALGEBRA I

DATE: 18/01/18 **TIME:** 2 PM -4 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

(a) (i)Define the term symmetric matrix and give an example (3mks)

(ii)Prove that the intersection of two subspaces is indeed a subspace (6 mks)

(b) Row reduce the matrix $A = \begin{bmatrix} 1 & 2 & -3 & 0 \\ 2 & 4 & -2 & 2 \\ 3 & 6 & -4 & 3 \end{bmatrix}$ to echelon form. (5mks)

(c) Given that the subspace $W \in \mathbb{R}^2$ of vectors of the form $w = (a, 2a)$. Prove that W is a subspace of $\mathbb{R}^2$. (6mks)

(d) Given the subspaces

$U = span\{(1,3,-2,2,3), (1,4,-3,4,2), (2,3,-1,-2,9)\}$

$W = span\{(1,3,0,2,1), (1,5,-6,6,3), (2,5,3,2,1)\}$

Determine the basis and the dimension of $U + W$ (6 mks)

(e) (i) Find the kernel of the linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^2$ defined by

$T(x) = Ax$ where $A = \begin{bmatrix} 1 & -1 & -2 \\ -1 & 2 & 3 \end{bmatrix}$ (4mks)

QUESTION TWO (20MARKS)

a) A linear transformation function $T: \mathbb{R}^2 \to \mathbb{R}^3$ is defined as

$$T(v) = A(x) = \begin{bmatrix} 3 & 0 \\ 2 & 1 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

i) Find $T(v)$ when $v = (2, -1)$

ii) Show that T is a linear transformation from $\mathbb{R}^2 \to \mathbb{R}^3$ (8mks)

b) Express the vector $v = (1, -2, 5)$ as linear combination of the vectors $u_1 = (1,1,1)$ $u_2 = (1,2,3)$ and $u_3 = (2, -1, 1)$ (5mks)

c) Find the dimension and the basis of the solution space W of the homogeneous system

$$x + 2y + 2z - s + 3t = 0$$
$$x + 2y + 3z + s + t = 0$$
$$3x + 6y + 8z + s + 5t = 0$$

(7mks)

QUESTION THREE (20 MARKS)

(a) Find the rank, a basis for the row space ,a basis for the column space and a basis for the null space of the matrix A given by $A = \begin{bmatrix} 1 & 2 & 1 & 3 & 1 & 2 \\ 2 & 5 & 5 & 6 & 4 & 5 \\ 3 & 7 & 6 & 11 & 6 & 9 \\ 1 & 5 & 10 & 8 & 9 & 9 \\ 2 & 6 & 0 & 11 & 9 & 12 \end{bmatrix}$ (9 mks)

(b) Determine whether the two subsets are subspaces of $\mathbb{R}^2$

i) The set of all points on the line $x + 6y = 0$ (6 mks)

ii) The set of points on the line $10x + 4y = 1$ (5 mks)

QUESTION FOUR (20 MARKS)

(a) Determine whether the set of vectors S in $\mathbb{R}^3$ is linearly independent or linearly dependent. Where, $S = \{(1,2,3) \ , \ (0,1,2) \ , \ (-2,0,1)\}$ (6mks)

(b) Express the solution set of the homogenous system

$$\begin{aligned} p - 2q + r - t &= 0 \\ 2p - 3q + 4r - 3t &= 0 \\ 3p - 5q + 5r - 4t &= 0 \\ -p + q - 3r + 2t &= 0 \end{aligned}$$

as a span of solution vectors . (8 mks)

c) Show that the vector $b = [5, \quad -6, \quad 2]$ is in the span of vectors $v = [1, \quad 1, \quad 1]$, $w = [1, \quad 1, \quad 0]$ and $u = [1, \quad 0, \quad 0]$ (6mks)

QUESTION FIVE (20 MARKS)

(a) Solve the system

$$\begin{aligned} x + 2y + 3z &= 1 \\ x + 3y + 6z &= 3 \\ 2x + 6y + 13z &= 5 \end{aligned}$$

using Gauss- Jordan elimination method (8mks)

(b) Find the basis for the kernel of T as a subspace of $\mathbb{R}^5$ if the linear transformation, $T : \mathbb{R}^5 \to \mathbb{R}^4$

and $T(x) = A(x)$ where $x \in \mathbb{R}^5$ and $A = \begin{bmatrix} 1 & 2 & 0 & 1 & -1 \\ 2 & 1 & 3 & 1 & 0 \\ -1 & 0 & -2 & 0 & 1 \\ 0 & 0 & 0 & 2 & 8 \end{bmatrix}$ (7 mks)

c) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{bmatrix}$ Using row operation method.

(5 mks)