(*Knowledge for Development*)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2016/2017 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: **MAT 121**

COURSE TITLE: **CALCULUS I**

DATE: 15/09/17 **TIME:** 8 AM -10 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of **3 Printed Pages**. Please Turn Over.

QUESTION ONE (30 MARKS)

a) Define the following terms;

i. Limit of a function (2mks)

ii. Continuity of a function (2mks)

b) Show that $\lim_{x\to 0} \frac{\sqrt{2x^2+9}-3}{2x^2} = \frac{1}{6}$ (3mks)

c) Find the derivative each of the following functions using any suitable method

i. $y = \cot x$ (3mks)

ii. $y = (x^3 + x^2 + 1)^4$ (3mks)

iii. $y = x^4 e^{2x}$ (2mks)

d) Find the equation of the tangent to the curve $x^2y + y^3x + 4x + 2y + 3 = 0$ at the Point (-3, 2) (6mks)

e) Given that $y = e^{2x} \sin x$, show that $y'' - 4y' + 5y = 0$ (3mks)

f) State Rolle's theorem and hencefind the value of C satisfying the conclusion of Rolle's Theorem for $f(x) = x^3 + 2x^2 - x - 1$ on the interval [-1, 0] (6mks)

QUESTION TWO (20 MARKS)

a) Evaluate the following limits;

i. $\lim_{x\to 4} \sin^{-1} \frac{2-x}{4-x^2}$ (4mks)

ii. $\lim_{x\to 2} \frac{x^2-4}{x-2}$ (3mks)

iii. $\lim_{x\to 1} \frac{x-1}{\sqrt{x}-1}$ (3mks)

b) Determine whether the following functions are continuous at the indicated points;

i. $h(x) = \begin{cases} \frac{x^3-1}{x-1} & x \neq 1 \\ 3 & x = 1 \end{cases}$ at $x = 1$ (4mks)

ii. $g(x) = \frac{x^2-4}{x-2}$ at $x = 2$ (3mks)

iii. $f(x) = \begin{cases} x^2 + 1 & x < 2 \\ 5 & x = 2 \\ -x + 7 & x > 2 \end{cases}$ at $x = 2$ (3mks)

QUESTION THREE (20 MARKS)

a) Sketch the curve of the function $y = x^2 - 5x + 6$ (7mks)

b) (i) Identify all the stationary points of the equation below

$$y = \frac{1}{4}x^4 + \frac{2}{3}x^3 - \frac{9}{2}x^2 - 18x \qquad (7mks)$$

(ii) Determine the nature of the stationary point (6mks)

QUESTION FOUR (20 MARKS)

a) Evaluate the derivative of the following functions using any suitable method, simplifying your answer where possible

i. $y = e^{2x} \ln x$ (1mks)

ii. $y = \operatorname{cosec} x$ (3mks)

iii. $y = \frac{x^2-2}{2x+1}$ (2mks)

b) Find the derivative of the following functions using the first principle

i. $y = x^2 + 2x + 8$ (4mks)

ii. $y = \cos x$ (6mks)

c) Given the function $y = \ln(x^2 + x)$, determine $\frac{d^2y}{dx^2}$ (4mks)

QUESTION FIVE (20 MARKS)

(a) A stone is projected vertically upwards and its height is S metres after t seconds is given by $S = 16t - 4t^2$. Find;

(i) the maximum height attained by the stone (3 mks)

(ii) the velocity when $t = 1$ (3 mks)

(iii) the acceleration when $t = 1$ (2 mks)

(b) A company that manufactures dog food wishes to pack the food in closed cylindrical tins, what should be the dimension of each tin if each is to have a volume of $128\pi cm^3$ and the minimum possible surface area? (6 mks)

(c) A square sheet of metal has an area of $100m^2$. An open rectangular tank is to be made by cutting equal squares from the corners and bending the sides up. Find the height of the tank for which the volume will be maximum. (6 mks)