(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2016/2017 ACADEMIC YEAR

FIRST YEAR THIRD SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: **MAT 121**

COURSE TITLE: **CALCULUS I**

DATE: 13/09/17 **TIME:** 2 PM -4 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

a) Obtain the derivative of the function for $f(x) = x^3 - 2x^2 + 4x$ using the 1^{st} principles. (4 mks)

b) Find the equation of the line that is tangent to the curve $2x^2 + xy - y^2 = 10$ at the point $(-2,3)$ (4 mks)

c) Determine the derivatives of the following

(i) $y = 2x^2 ln3x$ (3 mks)

(ii) $y = \left(\sqrt{t} - 4t\right)^3$ (4 mks)

(iii) $y = \frac{e^{-3x^2}}{1-cos3x}$ (4 mks)

d) Given the function $y = x^3 - 3x^2 + 2$

(i) Find all the stationary points on the curve (6 mks)

(ii) State the nature of the stationary points (3 mks)

e) State the Rolle's theorem (2 mks)

QUESTION TWO (20 MARKS)

a) Evaluate the limits

(i) $\lim_{x\to 1} \frac{x-1}{\sqrt{x}-1}$ (3 mks)

(ii) $\lim_{x\to 2} \frac{x^2-4}{x-2}$ (3 mks)

(iii) $\lim_{x\to 0} \frac{tan\beta x}{sin\alpha x}$ (5 mks)

b) Given that $x = 3cos\theta$ and $y = 3sin\theta$ where θ is a constant find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{2}$ (4 mks)

c) Joseph has 100 M of fencing wire which he can put against an already existing fence to form a rectangular pen to enclose his cows. What is the maximum area can he enclose (5 mks)

QUESTION THREE (20 MARKS)

a) Determine the equation of the normal line to the graph of $xy^2 + 7x + 4xy - 4 = 0$ at (1,-1) (4 mks)

b) Find $\frac{dy}{dx}$ given that $y = cosx^3$ (4 mks)

c) The distance S metres moved by a body in t seconds is given by $S = 2t^3 - 13t^2 + 24t - 12$. Find

(i) The velocity when $t = 3$ seconds (4 mks)

(ii) The time when the body is instantaneously at rest (4 mks)

(iii) The time when acceleration is $10\ m/s^2$ (4 mks)

QUESTION FOUR (20 MARKS)

a) Find $\frac{dy}{dx}$ given that $y = cot^3(2x^2 - 3x + 4)$ (4 mks)

b) Determine whether the limit of each of the following functions exist at indicated point $f(x) = \frac{e^{2x}}{x^2-1}$ at $x = 1$ (4 mks)

c) Determine whether the following function is continuous at $x = 2$

$$f(x) = \begin{cases} x^2 + 1 & x < 2 \\ 5 & x = 2 \\ -x + 5 & x > 2 \end{cases}$$ (5 mks

d) Sketch the curve $y = x^3 - 6x^2 + 9x + 17$ (7 mks)

QUESTION FIVE (20 MARKS)

a) Find y^{II} given that $y = \ln(2x^2 - x + 3)$ (5 mks)

b) If $y = \frac{sinx}{x^2}$ prove that $x^2y^{II} + 4xy^I + (x^2 + 2)y = 0$ (5 mks)

c) A piece of wire 8M long is cut into 2 parts. If the parts are bent to form a square and a circle respectively. Find the radius of the circle if the sum of their areas is minimum (5 mks)

d) Show that the slope of the line tangent to the curve $sinxy = x^2cosy$ at $\left(2, \frac{\pi}{2}\right)$ is $\frac{\pi}{4}$ (5 mks)