(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: MAT 121

COURSE TITLE: CALCULUS I

DATE: 05/10/18 **TIME: 8 AM -10 AM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

a) Given that $y = 3x^2 - 2x - 7$ find $\frac{dy}{dx}$ using the first principle. (4mks)

b) Determine the equation of the normal to the curve $y = x^2 + 4$ at the point $y = 20$ in the form $y = mx + c$ (5 mks)

c) Find the gradient of the curve $y^2 + 4xy + x^2 = 6$ at the point (1, 1) (4 mks)

d) Differentiate the following functions with respect to x. (6 mks)

i. $y = \sec x$

ii. $y = e^{2x} \ln x$

iii. $y = (x^3 - 4x)^{10}$

e) Evaluate $\lim_{x\to 1} \frac{x^3-6x^2+11x-6}{x^2-6x+5}$ (3 mks)

f) Find the turning points of the graph of the function$y = \frac{1}{3}x^3 - \frac{5}{2}x^2 + 6x$ and determine the nature of the points. (8 mks)

QUESTION TWO (20 MARKS)

(a) Evaluate the following limits

(i) $\lim_{x\to 4} \frac{x^2-2x-8}{x^2-4x}$ (3 mks)

(ii) $\lim_{x\to 1} \frac{\sqrt{x}-2}{x-1}$ (3 mks)

(b) Determine whether the following functions are continuous or notat the indicated points

(i) $f(x) = \frac{x^2-4}{x^3+1}$ at $x = 1$ (4 mks)

(ii) $f(x) = \frac{x^2-3x}{x^2-9}$ at $x = 3$ (5 mks)

(iii) $h(x) = \begin{cases} \frac{x^3-1}{x-1} & x \neq 1 \\ 3 & x = 1 \end{cases}$ at $x = 1$ (5 mks)

QUESTION THREE (20 MARKS)

a) Find $\frac{dy}{dx}$ given

i. $y = \sqrt[3]{x}$ (2mks)

ii. $y = \ln\sqrt{x^2 + 1}$ (3 mks)

b) Find the value of C satisfying the conclusion of Rolle's theorem for $f(x) = x^3 - 3x^2 + 2x + 2$in the interval [0, 1] (6 mks)

c) Using the first principle, find $\frac{dy}{dx}$ given that $y = \sin x$ (5mks)

d) If $y = e^{2x}$, show that $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$ (4 mks)

QUESTION FOUR (20 MARKS)

a) Determine the equation of the normal line to the graph of $xy^2 + 7x + 4xy - 4 = 0$ at (1,-1) (4 mks)

b) Find $\frac{dy}{dx}$ given that $y = cosx^3$ (4 mks)

c) The distance S metres moved by a body in t seconds is given by $S = 23 - 13t^2 + 20t - 12$. Find

(i) The velocity when $t = 2$ seconds (4 mks)

(ii) The time when the body is instantaneously at rest (4 mks)

(iii) The time when acceleration is $7\ m/s^2$ (4 mks)

QUESTION FIVE(20 MARKS)

(a) Define the term stationary points. (1 mk)

(b) Given the function $y = \frac{1}{4}x^4 + \frac{2}{3}x^3 - \frac{9}{2}x^2 - 18x$ find

(i) all the stationary points on the curve (9 mks)

(ii) state the nature of all the stationary points (6 mks)

(c) Sketch the curve of the function $y = x^2 - 6x + 8$ (4 mks)