(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: MAT 121

COURSE TITLE: CALCULUS I

DATE: 16/01/18 **TIME:** 9 AM -11 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

a) Define a limit of a function and hence evaluate; $\lim_{x\to 1}\frac{\sqrt{(x^3-1)(x-1)}}{x-1}$ (5 mks)

b) Find the derivative of the function $y = x^2 + 2x - 4$ using the first principle. (3 mks)

c) If $f(x) = \ln(x^2 + x)$, find $f''(2)$ (4 mks)

d) Using any suitable method, find the derivative of the function $y = \cos^2 x - 1$ (3 mks)

e) Determine the gradient of the curve $2x^2 + xy + y^3 = 0$ at the point (1, 2). (4 mks)

f) The distance of an object at any given time is given by $S = 5t^2 - t^3$. Calculate the time when acceleration is $-14ms^{-1}$. (4 mks)

g) (i)State Rolle's theorem and hence find the value of x_o prescribed in the theorem for $f(x) = x^2 - 5x + 4$ on the interval $1 \le x_o \le 4$ (5 mks)

(ii) Let $f(x) = x^2 - x$. Does Rolle's Theorem guarantee the existence of some C in (1, 2) with$f'(C) = 0$. Give a reason for your answer. (2 mks)

QUESTION TWO (20 MARKS)

a) Define the term continuity of a function (2 mks)

b) Determine if the functions given below are continuous at the indicated points

(i) $f(x) = \begin{cases} \frac{x^2-1}{x-1} & x \neq 1 \\ 2 & x = 1 \end{cases}$ at $x = 1$ (3 mks)

(ii) $g(x) = \begin{cases} x + 4 & x < 2 \\ 3 & x = 2 \\ x^2 + 2 & x > 2 \end{cases}$ at $x = 2$ (5 mks)

c) Using the definition of limits, show that $\lim_{x\to -\frac{3}{2}}(1 - 4x) = 7$ (4 mks)

d) Evaluate the limits

(i)$\lim_{x\to 1}\frac{x^4+x^3-x-1}{x-1}$ (3 mks)

(ii) $\lim_{x\to 1^-}\frac{(x-1)(x+1)}{|x-1|}$ (3 mks)

QUESTION THREE (20 MARKS)

a) Evaluate the derivatives of the following functions

(i) $y = (x^3 + x)^9$ (2 mks)

(ii) $y = \frac{x-1}{x^2+x}$ (2 mks)

(iii) $y = \ln\frac{1}{\sqrt{x^2+2x}}$ (3 mks)

b) Given that $y = e^{2x}\sin x$ show that $y'' - 4y' + 5y = 0$ (4 mks)

c) Find derivative of the function $y = \cos x$, using the first principle (5mks)

d) Determine the value of $\frac{d^2y}{dx^2}$ if $y = \frac{1}{x^2+1}$. Simplify your answer. (4 mks)

QUESTION FOUR (20 MARKS)

a) Definethe term a stationary point (2 mks)

b) Given that $y = 0.25x^4 + 0.5x^3 + 0.25x^2 + 4$, from the equation

i. Identify all the stationary points (6 mks)

ii. Determine the nature of the stationary points (6 mks)

iii. Sketch the curve (3 mks)

c) Sketch the curve of another curve with the equation $y = x^3 + 1$. (3 mks)

QUESTION FIVE (20 MARKS)

a) The position of a stone projected vertically upwards at any time is given by $S = 2t - t^3$. Find;

i. The time at which the object has zero velocity (3 mks)

ii. The max height of the stone from the ground. (2 mks)

iii. Acceleration of the stone at $t = 3$ (2 mks)

iv. The height above the ground from which the stone is projected. (2 mks)

b) Find the equation of a line perpendicular to the curve$y = x^3 - 8x + 4$ at the point (2, -4) in the form $y = mx + c$ (5 mks)

c) Find the base radius, **r** of a cylindrical hole with max volume which can be drilled into a cone of height 16 cm and radius 12 cm. **(Hint: use the diagram below)** (6 mks)