(10)

(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2015/2016 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER

MAIN EXAMINATION (SCHOOL BASED)

FOR THE DEGREE OF BACHELOR OF EDUCATION

COURSE CODE: MAT 121

COURSE TITLE: CALCULUS I

DATE: 18/4/16 **TIME:** 11.00 AM -1.00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

(a) State L Hospital's rule and hence evaluate the limit, $\lim_{x\to 0} \frac{e^{2x}-1}{x}$ (5 mks)

(b) Using the first principle show that if $y = \sin x$ then $\frac{dy}{dx} = \cos x$ (4 mks)

(c) Evaluate derivatives of the functions, simplify your answer.

(i) $y = \ln(x^2 + 2)^6$ (3 mks)

(ii) $y = \frac{x^2+3}{2x^2-4x}$ (3 mks)

(d) Find the gradient of the curve $xy^2 + y^2 - x^3 + 8 = 0$ at the point (1, 2) (4 mks)

(e) Find the equation of the of the normal to the curve $y = x^2 - 4x + 1$ at the point (2, -3) in the form $y = mx + c$ (5 mks)

(f) (i) State Rolle's theorem (2 mks)

(ii) find the value of C prescribed in Rolle's theorem for $y = x^3 + 2x^2 - x - 1$ on the interval $-2 < C < 1$ (4 mks)

QUESTION TWO (20 MARKS)

(a) What are stationary points. (1 mk)

(b) Given the function $y = \frac{1}{4}x^4 + \frac{2}{3}x^3 - \frac{9}{2}x^2 - 18x$ find

(i) all the stationary points on the curve (9 mks)

(ii) state the nature of all the stationary points (6 mks)

(c) Sketch the curve of the function $y = x^3 - 2x^2 - x + 2$ (4 mks)

QUESTION THREE (20 MARKS)

(a) Find the derivatives of each of the following functions using any appropriate method

(i) $y = (x^2 - 3x + 4)^3$ (3 mks)

(ii) $y = x \ln x^2 + 2$ (4 mks)

(b) Use quotient rule to find $\frac{dy}{dx}$ of $y = \frac{x^2+3}{2x+4}$ simplify your answer (4 mks)

(c) If $y = e^{2x}$, show that $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$ (5 mks)

(d) Find the derivative of $\tan x$ (4 mks)

QUESTION FOUR (20 MARKS)

(a) Evaluate the following limits

(i) $\lim_{x\to 4} \frac{x^2-2x-8}{x^2-4x}$ (3 mks)

(ii) $\lim_{x\to 1} \frac{\sqrt{x}-2}{x-1}$ (3 mks)

(b) Determine whether the following functions are continuous or notat the indicated points

(i) $f(x) = \frac{x^2-4}{x^3+1}$ at $x = 1$ (4 mks)

(ii) $f(x) = \frac{x^2-3x}{x^2-9}$ at $x = 3$ (5 mks)

(iii) $h(x) = \begin{cases} \frac{x^3-1}{x-1} & x \neq 1 \\ 3 & x = 1 \end{cases}$ at $x = 1$ (5 mks)

QUESTION FIVE (20 MARKS)

a) A stone is projected vertically upwards. If its height S after t seconds is given by $S = 10t - t^2$, find;

i. The maximum height attained by the stone. (3 mks)

ii. Its velocity at $t = 1$ (1 mk)

iii. Its acceleration at $t = 2$ (3 mks)

b) A piece of wire 18cm long is to be bent to form a rectangle. If its length is x cm. Find;

i. The expression for its area in terms of x (2 mks)

ii. The dimension of the rectangle with maximum area for the expression. (3 mks)

c) A company that manufactures dog food wishes to pack food in closed cylindrical tins. What should be the dimension of each tin if each is to have a volume of $128\pi cm^2$ and a minimum possible surface area. (8 mks)