(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: **MAT 110**

COURSE TITLE: **BASIC CALCULUS**

DATE: 08/10/18 **TIME:** **3 PM -5 PM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of **4 Printed Pages**. Please Turn Over.

QUESTION ONE (30marks)

(a). i. What is a function (1mk)

ii. Classify the following as functions or not

a). $x^2 + y = 1$ b). $y^2 = x$ Explain. (2 mks)

(b). i. Differentiate the function $f(x) = \cos x$from the first principal (4 mks)

ii. Differentiate $f(x) = \sqrt{x(3x^2 + 7)^3}$ (3 mks)

iii. If $y = \log_2(4 + 9x)$ what is $y'(0)$? (3 mks)

(c). i. Find the slope the function $f(x) = \frac{(x+2)^2}{(4-x^2)^3}$ at $x = -1.$ (4mks)

ii.Find the first derivative of$2y = x^2y + \cos x.$ (3 mks)

iii. Find the antiderivative of $y = \frac{1}{10x} + 4\sec(2x) - \sec^2(4x + \pi)$ (3mks)

(e).i. Evaluate the limit $\lim_{h\to 0} \frac{6x^2+(6x-h)^2-42x^2}{h}$ (3mks)

iii.Is the function $f(x)$ defined below continuous on the whole of the real axis? Show your working.$f(x) = \begin{cases} 1 - x & x \le 2 \\ x^2 - 2x & x > 2 \end{cases}$. (4mks)

QUESTION TWO (20MKS)

(a).Find $\frac{dy}{dx}$ (i).$y = \tan(e^{4x})\sec(e^{4x})$. (4mks)

(ii). $y = \ln(e^{4x} + 2x + 1)$ (3mks)

(b).Find the range of $f(x) = e^{2x^2-4}$ (2 mks)

(c).i.What is the precise definition of a limit. (2mks)

ii. Using the definition above, proof that $\lim_{x\to 3}(4x - 7) = 5$. (4 mks)

(d). If $f(x) = \frac{2x}{5x-3}$ and $g(x) = 3 - 3x$ find

(i). $f \circ g(x)$ (2 mks)

(ii) $g \circ f(1)$ (3 mks)

QUESTION THREE (20MKS)

(a). Given that $x(t) = t + 2\sin 2t$ and $y(t) = t + 2\cos 5t$, find

(i). $\frac{dy}{dx}$ at $x = \pi$. (2mks)

(ii). $\frac{d^2y}{dx^2}$ at $x = \pi$ (5mks)

(b). Integrate

(i). $\int x^2\sqrt{1 + x^3}\ dx$ (3 mks)

(ii). $\int \frac{x^3}{1-x^4}\ dx$(2 mks)

(c). Find the absolute maximum and absolute minimum values of $f(x)$ on the interval $[-3,2]$ if

$f(x) = x^4 - 8x^2 + 2$ (5 mks)

(d). If $f(x) = \frac{2}{x-4}$, show that $f \circ\ f^{-1}(x) = x$ if f^{-1} is the inverse function of f. (3 mks)

QUESTION FOUR (20MKS)

(a). i. State the Rolle's Theorem. (2 marks)

ii. Verify that the function $f(x) = \sin x + \cos x$ satisfies Rolle 's Theorem on $[0,2\pi]$ then find the number(s) c that satisfy the conclusion of the theorem. (5mks)

(b). A mass attached to a vertical spring has position function given by $y = A\sin\omega t$where A is the amplitude of oscillations and ω is a constant.

i. Find the velocity and acceleration as functions of time. (2 mks)

ii. Show that the acceleration is proportional to the displacement y. (2 mks)

iii. Show that the speed is maximum when acceleration is zero. (3 mks)

(c). Evaluate the limit

$\lim_{\theta\to\pi/3} \frac{\cos\theta - 0.5}{\theta - \pi/3}$ (3 mks) (d). Find the equation of the normal line to the curve $y = 4x^2 + 6x + 5$ at $(-1,3)$. (3 mks)

QUESTION FIVE (20MKS)

(a). When is a function continuous at a point? (2 mks)

(b). Given the function $f(x) = \begin{cases} -1 \; if \; x \leq -1 \\ 3x \; if \; -1 < x < 1 \\ 2x - 1 \; if \; x \geq 1 \end{cases}$

(i). Find $f(-1)$, $f(0)$ and $f(1)$ (3 mks)

(ii). Sketch $f(x)$ (2 mks)

(iii). Show that $f(x)$ is or is not continuous at $x = 1$. (4 mks)

(c). For what values of x does $f(x) = x + 2\sin x$ have a horizontal tangent? (3 mk)

(d). The position of a particle is given by the equation

$$s = -t^3 + 8t^2 + 16t$$

Where t is measured in seconds and s in meters.

(i). Find the velocity at time t. (1 mk)

(ii). What is the velocity after 2 s and 5 s respectively? (2 mks)

(iii). When is the particle momentarily at rest? (3 mk)