(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: MAT 110

COURSE TITLE: BASIC CALCULUS I

DATE: 18/01/18 **TIME:** 2 PM -4 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30marks)

(a). i. What are monotonically increasing and decreasing functions? (2 mk)

ii. Identify the following equations as either functions or not

a). $x^2 + y^2 = 36$ b). $x = \pm\sqrt{y}$, $y > 0$. Explain. (2 mks)

iii. Find the domain of $f(x) = \frac{\sqrt{2x+4}}{x^2}$ (2 mks)

(b). i. When is the $f(x)$ differentiable at $x = a$. (1 mk)

ii. Given that $f(x) = \frac{2x+1}{1-6x}$ is differentiable on some interval I determine its derivative from the first principle. (3 mks)

(c). i. Find the slope the function $f(x) = (x+2)^2(4-x)^3$at $x = -1$. (3 mks)

ii.Find $y'(x)$ if $\ln y = xy + \sin x$. (3 mks)

iii. If $y = \sin\left(e^{x^2+1}\right)$ find $\frac{dy}{dx}$. (3 mks)

(d). i.$ax - by = 1$ is the equation of the tangent of the function $y = x \ln x^2$ at $x = e$.

Find the value of b. (3 mks)

ii. Find the antiderivative of $y = \frac{12x^3-5x^2+1}{x^2}$ (2 mks)

(e). i.What is a limit of a function? (1 mk)

ii. Find the limit of $\lim_{x\to 1} f(x)$ if $f(x) = \begin{cases} 7-4x & x < 1 \\ x^2+2 & x \geq 1 \end{cases}$. (3 mk)

iii. Evaluate the limit $\lim_{z\to 8} \frac{2z^2-17z+8}{z-8}$ (2 mks)

QUESTION TWO (20MKS)

(a). Find $y''(x)$ if $\tan(x+y) = x^2$ (4 mks)

(b). Simplify (i). $\int x^2(3-x^3)dx$ (3 mks)

(ii). $\int x^3 \cos(x^4+2)\, dx$ (4 mks)

(c). Differentiate $h(x) = \frac{\cos x}{\tan 2x}$ (3 mks)

(d). Evaluate the limit$\lim_{x\to 0} \frac{5x^3-2\sin 2x}{8x}$ (2 mks) (e). A manufacturer sells ready meat in closed cylindrical cans each holding $128\pi\ cm^3$.

(i). Find an expression showing the surface area of each tin. (1 mks)

(ii). Find the lowest possible dimension of the each tin. (3 mks)

QUESTION THREE (20MKS)

(a). Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ given that

$x(t) = t^2 - t$ and $y(t) = \frac{2}{3}t^3 + \frac{1}{2}t^2 - t.$ (5 mks)

(b). Find the slope of $x + 2y = (x - y^2 + 1)^2$at $(1, -1)$. (3 mks)

(c). Find all stationery points of the curve $y = \frac{1}{4}x^4 - \frac{2}{3}x^3 - \frac{9}{2}x^2 + 18x + 6$ and state their nature and sketch the curve (8mks)

(d). Given that $f(x) = \frac{5}{x-7}$ find$\lim_{x\to7^+} f(x)$, $\lim_{x\to7^-} f(x)$ and $\lim_{x\to7} f(x)$. (4 mks)

QUESTION FOUR (20MKS)

(a). i. State the Rolle's Theorem. (2 marks)

ii. Determine whether the function $f(x) = 3\sin 2x$ satisfies the conditions of Rolle's Theorem for the interval $[0, 2\pi]$. If so, find all numbers c that satisfies the conclusion of the theorem. (4 mks)

(b). A farmer wishes to enclose a rectangular field with $500\ ft$ of fence material where one of the longest side of the field is a building. Determine the dimensions that will maximize the enclosed area. (4 mks)

(c). Show that differentiability at a point, implies continuity at the same point. (4 mks)

(d). If $f(x) = \frac{x}{x+3}$ and $g(x) = 4x - 1$

(i). Find $f \circ g$ (2 mks)

(ii). Show that $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$ (4 mks)

QUESTION FIVE (20MKS)

(a). i. Differentiate between odd and even function. (2 mks)

ii. Classify the following functions as odd, even or neither. Show your working. (2 mks)

(a). $y = -3x^2 + 1$ (b) $y = x^3 + x^2 + 1$

(a). Find the derivative of the following functions

(i). $f(x) = \left(\frac{x-2}{2x+1}\right)^9$ (4 mks)

(ii). $f(x) = \frac{2x-1}{x^2(3x+5)^9}$ (5 mks)

(c). i. When is a function $f(x)$ continuous at $x = c$? (1 mks)

ii. Let $f(x)$ be the function defined below.

$$f(x) = \begin{cases} \dfrac{2x^2 - 3x - 2}{x - 2} & \text{for } x \neq 2 \\ k & \text{for } x = 2 \end{cases}$$

For which value of k is $f(x)$ continuous at $x = 2$. (4 mks)

(d). Determine the slope of the normal line to the function $y = \frac{8}{x^2} + 4x + 9$ at $x = 2$.

(2 mks)