(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2016/2017 ACADEMIC YEAR

FIRST YEAR SECOND SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: **MAT 104**

COURSE TITLE: ALGEBRAIC STRUCTURES I

DATE: 29/09/17 **TIME:** 11.30 AM -1.30 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

a. Define the following

i. Trivial subgroup (1mark)

ii. Subgroup generated by x (2marks)

iii. Bijective function (2 marks)

iv. Inverse of a function (2 marks)

v. Union of sets (2 marks)

b. Give four examples of fields (4 marks)

c. Let $\Omega = \{1,2,2 \ldots \ldots \ldots . n$ and let s_n denote the set of all permutation of Ω .

Show that s_n is a group under composition of elements (4 marks)

d. If S is a subset of the group G, show that s is a subgroup of G if and only if S is nonempty and whenever $a, b\epsilon S$,then $ab^{-1}\epsilon S$ (4 marks)

e. If A is an invertible matrix, show that its inverse is unique (5 marks)

f. For the matrix

$$A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

Verify that $x_1 = (-3 \quad -1 \quad 1)$ and $x_2 = (1 \quad 0 \quad 0)$ are eigenvectors of A and find their corresponding eigenvalues (4 marks)

QUESTION TWO (20 MARKS)

a. Define the following

i. Binary operation (2 marks)

ii. Group (3 marks)

b.

i. Determine the group of symmetries of D_4 (8 Marks)

ii. What is the order of D_4 (1 Mark)

c. Determine the symmetric group S_3 (7marks).

QUESTION THREE (20 MARKS)

a. Define the following

i. Subgroup (2 marks)

ii. Cyclic subgroup (2marks)

iii. Lagranges theorem (2 marks)

iv. Coset (2 marks)

b. Let $Z_{15} \geq \langle 5 \rangle = \{0,5.10\}$ for $k \epsilon Z_{15}$, and the left coset is k+<5>,determine 5 distinct cosets of <5> in Z_{15} (5 marks)

c. Show that G is cyclic if $|G| = p$ is a prime (3 marks)

d. In a field ,show that a product of two nonzero elements is nonzero (2 marks)

e. If a, b, c are elements of a field and $a \neq 0$, show that the following cancellation law holds $ab = ac \Rightarrow b = c$ (2 marks)

QUESTION FOUR (20 MARKS)

a. Define the following

i. Ring (2 marks)

ii. Field (3 marks)

b. State three properties of rings (3 marks)

c. Show that Z_4 is not a field (3 marks)

d. Construct a cayley table for multiplication in Z_6 (3 Marks)

e. Show that cosets are either identical or disjoint (4 marks)

QUESTION FIVE (20 MARKS)

a. Find the difference. Write the answer in standard form.

i. $(4x^2 - 3) - (2x^2 + 6)$ (2 marks)

ii. $(-3x^3 + 7) - (5x^3 - x^3)$ (2 marks)

b. Define the following

i. Solving binomial equations (1 mark)

ii. Circulant matrices (1 Mark)

c. Generate a 3×3 circulant matrix starting with $[a, b, c]$ (3 marks)

d. Consider the circulant matrix

$$C = \begin{bmatrix} 1 & 2 & 1 & 3 \\ 3 & 1 & 2 & 1 \\ 2 & 1 & 3 & 1 \end{bmatrix}$$

i. Read the polynomial q from the first row of C. (1 mark)

ii. With n=4, determine the nth roots of unity (2 marks)

iii. Compute the eigenvalues of C. (4 marks)

e. Compute the corresponding eigenvectors (4 marks)