



(Knowledge for Development)

KIBABII UNIVERSITY
UNIVERSITY EXAMINATIONS
2017/2018 ACADEMIC YEAR
FIRST YEAR SECOND SEMESTER
SPECIAL/ SUPPLEMENTARY EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE
MATHEMATICS

COURSE CODE: MAT 104

COURSE TITLE: ALGEBRAIC STRUCTURES I

DATE: 19/10/18

TIME: 11.30 AM -1.30 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 4 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

- a.
- i. Determine the group of symmetries of D_4 (8 Marks)
 - ii. What is the order of D_4 (1 Mark)
- b. Let $\Omega = \{1, 2, 3, \dots, n\}$ and let S_n denote the set of all permutations of Ω .
Show that S_n is a group under composition of elements (4 marks)
- c. Show that cosets are either identical or disjoint (4 marks)
- d. Show that G is cyclic if $|G| = p$ is a prime (3 marks)
- e. Define the following
- i. Subgroup (2 marks)
 - ii. Cyclic subgroup (2 marks)
 - iii. Lagrange's theorem (2 marks)
 - iv. Coset (2 marks)
 - v. Group (3 marks)

QUESTION TWO (20 MARKS)

- a. Let $Z_{15} \cong \langle 5 \rangle = \{0, 5, 10\}$ for $k \in Z_{15}$, and the left coset is $k + \langle 5 \rangle$, determine 5 distinct cosets of $\langle 5 \rangle$ in Z_{15} (5 marks)
- b. Determine the symmetric group S_3 (7 marks).
- c. Define the following
- i. Bijective function (2 marks)
 - ii. Inverse of a function (2 marks)
 - iii. Union of sets (2 marks)
 - iv. Binary operation (2 marks)

QUESTION THREE (20 MARKS)

- a. State three properties of rings (3 marks)
- b. Show that Z_4 is not a field (3 marks)
- c. Construct a Cayley table for multiplication in Z_6 (3 Marks)
- d. In a field, show that a product of two nonzero elements is nonzero (2 marks)
- e. If a, b, c are elements of a field and $a \neq 0$, show that the following cancellation law holds $ab = ac \implies b = c$ (2 marks)
- f. Define the following
- i. Ring (2 marks)
- ii. Field (3 marks)

QUESTION FOUR (20 MARKS)

- a. Find the difference. Write the answer in standard form.
- i. $(4x^2 - 3) - (2x^2 + 6)$ (3 marks)
- ii. $(-3x^3 + 7) - (5x^3 - x^3)$ (2 marks)
- b. Define the following
- i. Solving binomial equations (1 mark)
- ii. Circulant matrices (1 Mark)
- c. Generate a 3×3 circulant matrix starting with $[a, b, c]$ (3 marks)
- d. Consider the circulant matrix
- $$C = \begin{pmatrix} 1 & 2 & 1 & 3 \\ 3 & 1 & 2 & 1 \\ 2 & 1 & 3 & 1 \end{pmatrix}$$
- i. Read the polynomial q from the first row of C . (1 mark)
- ii. With $n=4$, determine the n th roots of unity (2 marks)
- iii. Compute the eigenvalues of C . (4 marks)
- e. Compute the corresponding eigenvectors (4 marks)

QUESTION FIVE (20 MARKS)

- a. If S is a subset of the group G , show that S is a subgroup of G if and only if S is nonempty and whenever $a, b \in S$, then $ab^{-1} \in S$ (4 marks)
- b. If A is an invertible matrix, show that its inverse is unique (5 marks)
- c. For the matrix

$$A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

Verify that $x_1 = (-3 \ -1 \ 1)$ and $x_2 = (1 \ 0 \ 0)$ are eigenvectors of A and find their corresponding eigenvalues (4 marks)

- d. Define the following
- i. Trivial subgroup (1mark)
- ii. Subgroup generated by x (2marks)
- e. Give four examples of fields (4 marks)