(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

FIRST YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: MAT 104

COURSE TITLE: ALGEBRAIC STRUCTURES I

DATE: 10/08/18 **TIME:** 9 AM -11 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 4 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

a. Define the following

   i. Domain of function (2 marks)

   ii. Range of a function (2 marks)

   iii. Binary operation (2 marks)

   iv. Group (3 marks)

   v. Commutative (1mark)

b. Let G= [a_1, a_2... a_n] be a finite group. Show that any row and column of the multiplication table of G, each element of G appears exactly once (5marks)

c. Using the symmetries of a square, determine the cyclic group of a square (5marks)

d. Show that a cyclic group is always abelian (5marks)

e. Show that $(\mathbb{Z},+)$ is a group (5marks)

QUESTION TWO (20 MARKS)

a. Define the following

   i. Subgroup (2 marks)

   ii. the subgroup criterion (2marks)

   iii. abelian group (2 marks)

   iv. trivial subgroup (1 marks)

b. Let G be a group, $H \leq G$ and $K \leq G$. Show that $H \cap K \leq G$ (6 marks)

c. Let G be a group, $a \in G$. Show that $\langle a \rangle$ is a subgroup of G (4 marks)

d. Determine klein-4 group (3marks)

QUESTION THREE (20 MARKS)

a. Define the following

   i. Dihedral group (2 marks)

   ii. Lagranges theorem (2 marks)

   iii. The symmetric group (2 marks)

b. Determine the group of symmetries of D_4 (8 marks)

c. Determine the symmetric group S_3 (6 marks)

QUESTION FOUR (20 MARKS)

a. Define the following

iv. Coset (2 marks)

v. Order of a group (2 marks)

vi. Index of a group (2 marks)

b. Let $H \leq G$ and $x, y \in G$. Show that either $xH = yH$ or $xH \cap yH = \emptyset$ (7 marks)

c. Let H be a finite subgroup of a group G. Show that $|gH| = |H|$ for all $g \in G$(4marks)

d. If a, b, c are elements of a field and $a \neq 0$,show that the following cancellation law holds

$ab = ac \Rightarrow b = c$ (3marks)

QUESTION FIVE (20 MARKS)

a. Define the following

i. Ring (3marks)

ii. Field (2marks)

b. State three properties of rings (4 marks)

c. In the ring $\mathbb{Z}_3 = \{0,1,2\}$,show that every nonzero element is its own inverse (3 marks)

d. Give four examples of fields (4 marks)

e. Show that in a field, a product of two nonzero elements is nonzero or equivalently

$$ab = 0 \Rightarrow a = 0 or\ b = 0$$

f. Show that the element 2 has no inverse in the ring $\mathbb{Z}_4 = \{0,1,2,3\}$ (4marks)