300

(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2015/2016 ACADEMIC YEAR

FIRST YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: MAT 102

COURSE TITLE: FOUNDATION MATHEMATICS II

DATE: 10/5/16 **TIME:** 2 PM -4 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE COMPULSORY (30 MARKS)

(a) With the aid of examples briefly define the following terms (2mks)

(i) A vector

(ii) A matrix

(b) Find the angle between the vectors $\boldsymbol{a} = 2i + 3j - 3k$ and $\boldsymbol{b} = -4i + 5j + k$ (4 mks)

(c) If θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b} 0 \leq \theta \leq \pi$ show that $\|\boldsymbol{a} \times \boldsymbol{b}\| = \|a\|\|b\| sin\theta$ (5 mks)

(d) Show that $\vec{A} \times \vec{B}$ is a vector orthogonal to both $\vec{A}$ and $\vec{B}$ (5mks)

(e) Reduce the system into row-echelon form hence by back substitution solve it (5 mks)

$$x + y + z = 2$$
$$-x + 3y + 2z = 8$$
$$4x + 5y + z = 6$$

(f) Find all the solutions of the following system of equations using augmented matrix (5 mks)

$$3x + 2y - 3z = -9$$
$$x - 3y + 2z = 4$$
$$-2x + 5y - 5z = -17$$

(g) Find A if $(A^T + 3I)^{-1} = \begin{bmatrix} 3 & 2 \\ -2 & 0 \end{bmatrix}$ (4 mks)

QUESTION TWO (20 MARKS)

(a) Prove that $\vec{A}.(\vec{B} + \vec{C}) = \vec{A}.\vec{B} + \vec{A}.\vec{C}$ (6 mks)

(b) Find the projection of $2i - 3j + 2k$ on $5i - 3k$ (4 mks)

(c) Calculate $\vec{P} \times \vec{Q}$ given $\vec{P} = \langle 2,3,4 \rangle$ and $\vec{Q} = \langle -3,-1,-5 \rangle$ (4 mks)

(d) Compute the rank of

$$A = \begin{bmatrix} 1 & 3 & -2 & 6 \\ 2 & 5 & 3 & 0 \\ -3 & 4 & 1 & 7 \end{bmatrix}$$ (6 mks)

QUESTION THREE(20 MARKS)

(a) Find the value of μ if $\vec{P} = 2\mu i + 5j + 9k$ and $\vec{Q} = \mu i + 3\mu j + 2k$ are perpendicular (4 mks)

(b) Find the cofactors of matrix

$$A = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 3 & 4 \\ 1 & -2 & 6 \end{bmatrix}$$ (6 mks)

(c) Use adjoint method to find the inverse of the matrix

$$C = \begin{bmatrix} 2 & 0 & 7 \\ 0 & 5 & 6 \\ 1 & 3 & 4 \end{bmatrix}$$ (10 mks)

QUESTION FOUR(20 MARKS)

(a) Determine if the two vectors are parallel, orthogonal or neither (3 mks)
$\langle 5, -1, 3\rangle$ and $\langle 2, 4, -2\rangle$

(b) Prove the following characteristics of determinant using the matrices $A = \begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ -1 & 5 \end{bmatrix}$

(i) $\det(AB) = \det(A)\det(B)$ (5mks)

(ii) $det(B)^T = \det B$ (3mks)

(c) Find the values of b for which $\det A = 0$ $A = \begin{bmatrix} 1 & b & b \\ b & 1 & b \\ b & b & 1 \end{bmatrix}$ (5mks)

(d) If $P = \begin{bmatrix} 2 & 3 & -14 \\ 4 & 2 & 7\ 8 \\ 3 & -2 & 10\ 0 \end{bmatrix}$ and $Q = [2 \quad -3 \quad 1 \quad -2]$ find matrix R where

$$R = P \times Q^T$$ (4mks)

QUESTION FIVE(20 MARKS)

(a) Find the values of x_1, x_2 and x_3 using Cramer's Rule (10 mks)

$$3x_1 - 4x_2 - 2x_3 = 1$$
$$2x_1 + 5x_2 - 2x_3 = 3$$
$$x_1 + 2x_2 + x_3 = 2$$

(b) Find the value of x such that $(2x \quad 3x)\begin{pmatrix} 5x \\ 1 \end{pmatrix} = (18)$ (4 mks)

(c) Given the matrix $A = \begin{bmatrix} 1 & 3 & 2 \\ 5 & 6 & 4 \\ 8 & 7 & 9 \end{bmatrix}$ find det(adjoint A) (6 mks)