(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

FIRST YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: **MAT 102**

COURSE TITLE: FOUNDATION MATHEMATICS II

DATE: 15/01/18 **TIME:** 2 PM -4 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

Question One (30 MARKS)

(a) i What is the difference between a vector and a scalar? (2mks)

ii. Prove the following properties of vector addition:

$u + v = v + u$ (commutative property) (2mks)

$c(u + v) = cu + cv$ (distributive property) (2mks)

iii. Sketch the position vector $\overrightarrow{OB} = (0,5,-1)$ in R^3 (2mks)

(b) State and prove the Cauchy-Schwarz Inequality theorem (6mks)

(c) Solve the following system of linear equations

$$x_1 - 2x_2 + 4x_3 = 12$$
$$2x_1 - x_2 + 5x_3 = 18$$
$$-x_1 + 3x_2 - 3x_3 = -8$$

(5mks)

(d) i. If $A = \begin{bmatrix} 4 & 1 \\ 3 & 1 \end{bmatrix}$; compute $(A^t)^{-1}$ (3mks)

ii. Verify that $(A^{-1})^{-1} = A$ and $(AB)^{-1} = B^{-1}A^{-1}$ (4mks)

(e) Evaluate the determinant of the following matrix: (4mks)

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 4 & 2 & 1 \end{bmatrix}$$

Question Two (20 MARKS)

(a) Solve using Gauss-Jordan elimination with matrices:

$4x_1 + 8x_2 - 12x_3 = 44$

$3x_1 + 6x_2 - 8x_3 = 32$

$-2x_1 - x_2 = -7$

(7mks)

(b) Define the following terms:

i. Homogeneous system of linear equations
ii. A diagonal matrix
iii. An identity matrix
iv. Symmetric matrix

(4mks)

(c) i. Differentiate between minor and cofactor of determinants (2mks)

ii. Solve the following system of equations using Cramer's Rule

$x_1 + 3x_2 + x_3 = -2$

$2x_1 + 5x_2 + x_3 = -5$

$x_1 + 2x_2 + 3x_3 = 6$

(7mks)

Question Three (20 MARKS)

(a) Define

i. Unit vector
ii. Orthogonal vector

(2mks)

(b) Determine a unit vector perpendicular to the plane of a and b where a = 4i+3j-k and b = 2i-6j-3k. Also obtain the angle between a and b (6mks)

(c) Determine the inverse of the matrix A using elimination method

$$A = \begin{bmatrix} 1 & -1 & -2 \\ 2 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$$

(7mks)

(d) Compute linear combination 2A + 3B – 5Cgiven that

$$A = \begin{bmatrix} 1 & 3 \\ -4 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & -7 \\ 2 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & 2 \\ 3 & 1 \end{bmatrix}$$

Compute the product ABC of

$$A = \begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}; \quad B = \begin{bmatrix} 0 & 1 & 3 \\ -1 & 0 & -2 \end{bmatrix}; \text{ and } \quad C = \begin{bmatrix} 4 \\ -1 \\ 0 \end{bmatrix}$$

(5mks)

Question Four (20 MARKS)

a. Determine the trace of the matrix

$$A = \begin{bmatrix} 4 & 1 & -2 \\ 2 & -5 & 6 \\ 7 & 3 & 0 \end{bmatrix}$$

(2mks)

b. Determine the inverse of the matrix, if it exists

$$A = \begin{bmatrix} 1 & 1 & 5 \\ 1 & 2 & 7 \\ 2 & -1 & 4 \end{bmatrix}$$

(5mks)

c. Solve

$$2x_1 + x_2 + x_3 = 1$$
$$-x_1 + 2x_2 - 3x_3 = 3$$
$$x_1 + 3x_2 - 2x_3 = 4$$

(7mks)

d. i. Determine the distance between the points:

$$x = (1,-2,3,0) \text{ and } y = (4,0,-3,5) \text{ in } R^4$$

(2mks)

ii. Prove the triangle inequality $||u+v|| \leq ||u|| + ||v||$ (4mks)

Question Five (20 MARKS)

a. Prove that

i. u.v = v.u

ii. u.u≥ 0 and u.u = 0 iff u = 0

(6mks)

b. Determine the angle between the vectors u = (1,0,0) and v = (1,0,1) in R^3

(4mks)

c. If a x b = c x d and a x c = b x d, show that a.d and b.c are parallel vectors

(3mks)

d. Determine a unit vector perpendicular to the plane of a and b where a = 4i+3j-k and b = 2i-6j-3k. Obtain the angle between a and b

(7mks)