(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

FIRST YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: **MAT 102**

COURSE TITLE: FOUNDATION MATHEMATICS II

DATE: 07/08/18 **TIME:** 2 PM - 4 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

(a) Define the following terms (2 mks)
   (i) A matrix
   (ii) Rank

(b) Given the matrices $\boldsymbol{p} = 5j - 8k$ and $\boldsymbol{q} = -2i + j - 6k$
find the projection of $\boldsymbol{q}$ on $\boldsymbol{p}$ (4 mks)

(c) If $\boldsymbol{a} = 3i + 6j - 4k$ and $\boldsymbol{b} = 2i - j + 3k$ evaluate $-2(\boldsymbol{a} \times \boldsymbol{b}).\boldsymbol{a}$ (5 mks)

(d) Find K if $(4I - P^T)^{-1} = \begin{bmatrix} 0 & 1 \\ -2 & -4 \end{bmatrix}$ (5 mks)

(e) Given that and $A = \begin{bmatrix} 4 & -5 \\ 3 & -2 \\ 19 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 7 & 4 & -5 \\ 3 & 10 & 4 \end{bmatrix}$ find $(A^T - B)^T$ (3 mks)

(f) Given $A = \begin{bmatrix} -2 & -3 & -1 \\ -3 & -3 & -1 \\ -2 & -4 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ -6 & 2 & 3 \end{bmatrix}$ show that B is the inverse of A (4 mks)

(g) Find the solution of the following system of linear equations using augmented matrices (7 mks)

$$3x_1 + 2x_2 + 2x_3 + 4 = 0$$

$$4x_1 - x_3 + 2x_2 = 10$$

$$3x_1 + x_2 - 2x_3 - 5 = 0$$

QUESTION TWO (20 MARKS)

(a) Find the values of β for which $detM = 0$

$M = \begin{bmatrix} \beta & 1 & \beta \\ 1 & \beta & \beta \\ 1 & \beta & 1 \end{bmatrix}$ (6 mks)

(b) If $detA = -11$ and $\det B = 8$ calculate $det(A^3B^{-1}A^TB^2)$ given that A Is any square matrix (4 mks)

(c) Determine if the two vectors are parallel, orthogonal or neither
$4i - 2j + 3k$ and $i - 4j - 2k$ (4 mks)

(d) Compute the rank of $\begin{bmatrix} 1 & -3 & 1 & -1 & 0 & -1 \\ -1 & 3 & 0 & 3 & 1 & 3 \\ 2 & -6 & 3 & 0 & -1 & 2 \\ -1 & 3 & 1 & 5 & 1 & 6 \end{bmatrix}$ (6 mks)

QUESTION THREE (20 MARKS)

(a) Use Cramer's rule to find x, y and z (10 mks)

$$-5x - y + z = 6$$

$$-9x - y + 2z = 1$$

$$4x - 2y + 3z = 2$$

(b) Evaluate $detA$ given that $A = \begin{bmatrix} x & p-x & -0.5a \\ -y & q+y & -0.5b \\ z & r-z & -0.5c \end{bmatrix}$ and $det\begin{vmatrix} a & p & x \\ b & q & -y \\ c & r & z \end{vmatrix} = 10$

(5 mks)

(c) Solve the linear system of equations (5 mks)

$$-x_1 + x_2 + 3x_3 = 0$$
$$x_1 + 2x_2 + 3x_3 = 0$$

QUESTION FOUR (20 MARKS)

(a) Find the inverse of the matrix $\begin{bmatrix} 1 & 4 & 0 \\ -1 & 0 & -2 \\ 6 & -2 & -3 \end{bmatrix}$ and use it to solve the linear system of equations

$$x_1 + 4x_2 = 2$$

$$-x_1 - 2x_3 = 7$$

$$6x_1 - 2x_2 - 3x_3 = 13$$ (12 mks)

(b) Compute the adjoint of P given

$$P = \begin{bmatrix} 2 & 4 & -1 \\ -1 & 2 & 5 \\ 2 & -6 & 9 \end{bmatrix}$$ (8 mks)

QUESTION FIVE (20 MARKS)

(a) Find the angle between the vectors $-3i - 2j + 2k$ and $5i - 3k$ (4 mks)

(b) Show that $\|a \times b\| = \|a\|\|b\|sin\theta$ (5 mks)

(c) Reduce the system into row-echelon form hence by backward substitution solve it

$$-x + 2y + z = 1$$ (5 mks)
$$x + 3y + 2z = 7$$

$$2x + 5y + z = 3$$

(d) Given that $A = \begin{bmatrix} -1 & 5 \\ 4 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 5 \\ 2 & -3 \end{bmatrix}$

Prove that $\det(AB) = \det A \det B$ (6 mks)