



100

(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2016/2017 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: MAT 100

COURSE TITLE: MATHEMATICS FOR TECHNOLOGISTS

DATE: 15/09/17

TIME: 8 AM -10 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 4 Printed Pages. Please Turn Over.

QUESTION 1 (30 MARKS)

- a) By using the definition of a limit, prove that

$$\lim_{x \rightarrow 3} (2x + 1) = 7$$

(5 marks)

- b) Find the following limits if they exist:

i)
$$\lim_{x \rightarrow -\infty} \frac{3x^2 - 7}{2x + 7}$$

(3 marks)

ii)
$$\lim_{h \rightarrow 0} \frac{7(x+h)^2 - 7x^2}{h}$$

(3 marks)

- c) Find the derivative of the implicit function

$$7y^2 + \sin(3x) = 12 - y^4$$

(3 marks)

- d) Find the equation of the tangent line at the given point.

$$y^2 e^{2x} = 3y + x^2 \text{ at } (0, 3)$$

(4 marks)

- e) Evaluate the following

$$\int x^2 \cos(4x) dx$$

(5 marks)

- f) Determine the angle between vectors
- $\mathbf{a} = \langle 2, 2, 1 \rangle$
- and
- $\mathbf{b} = \langle 5, 3, 2 \rangle$

(3 marks)

- g) Determine whether the function
- f
- given by
- $f(x) = \frac{x^6 - x^2}{x + x^3}$
- is even, odd or neither.

(4 marks)

QUESTION 2 (20 MARKS)

- a) Find the unit vector that points in the direction of
- $\mathbf{w} = \langle -5, 2, 1 \rangle$

(2 marks)

- b) Given that
- $\vec{a} = \langle 2, 1, -1 \rangle$
- and
- $\vec{b} = \langle -3, 4, 1 \rangle$
- compute each of the following

i) $\vec{a} \times \vec{b}$

(3 marks)

ii) $\vec{b} \times \vec{a}$

(3 marks)

- c) Prove that
- $\vec{a} \cdot \vec{b} = \|\vec{a}\| \|\vec{b}\| \cos \theta$

(5 marks)

- d) Show that
- $\mathbf{u} \times \mathbf{u} = \mathbf{0}$
- for any vector
- \mathbf{u}
- .

(3 marks)

- e) A function
- f
- given by
- $f(x) = \frac{72}{ax + b}$
- is such that
- $f(7) = 9$
- and
- $f(b) = 12$
- . Find the values of
- a
- and
- b
- .

(4 marks)

QUESTION 3 (20 MARKS)

a) Find the derivative indicated in the following:

i) $\frac{d}{dx}[2\sin(3x + \tan(x))]$ (3 marks)

ii) $\frac{d}{dt}\left[\left(1 + \sqrt{t^3}\right)\left(t^{-3} - 2\sqrt[3]{t}\right)\right]$ (3 marks)

b) If $y = 3x^2 + 5x - 10$, show that $y - y'x + \frac{1}{2}y''x^2 + 10 = 0$ (4 marks)

c) 100 students were asked whether they had taken courses in any of the three areas; Sociology, Anthropology and History. The results were: 45 had taken sociology, 38 had taken Anthropology, 21 had taken History, 18 had taken both Anthropology and Sociology, 9 had taken Sociology and history, 4 had taken History and Anthropology while 23 had taken no courses in any of the three areas.

i) Draw a Venn-diagram to represent the above information. (6 marks)

Hence, determine the number of students who had taken courses in

ii) All the three areas (2 marks)

iii) Exactly one of the areas (1 mark)

iv) Exactly two of the areas (1 mark)

QUESTION 4 (20 MARKS)

a) Compute the determinant of $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 1 & 1 \\ -1 & 3 & 0 \end{pmatrix}$ (5 marks)

b) Show that the matrix $\begin{pmatrix} 1 & 2 & 1 & 1 \\ 2 & 3 & 0 & 5 \\ 3 & 5 & 1 & 4 \end{pmatrix}$ has rank 3. (2 marks)

c) If $\det A = 2$ and $\det B = 5$, calculate $\det(A^3 B^{-1} A^T B^2)$. (5 marks)

d) Determine the particular solution of the differential equation $5\frac{dy}{dx} + 2x = 3$, given the boundary conditions $y = 1\frac{2}{5}$ when $x = 2$. (4 marks)

e) The bending moment M of a beam is given by $\frac{dm}{dx} = -w(l - x)$, where w and l are constants. Determine M in terms of x given that $M = \frac{1}{2}wl^2$ when $x = 0$. (4 marks)

QUESTION 5 (20 MARKS)

a) Perform the following integrals

i) $\int \sin^3\left(\frac{2}{3}x\right)\cos^4\left(\frac{2}{3}x\right)dx$ (6 marks)

ii) $\int \frac{3x+1}{x^2-x-6}dx$ (5 marks)

iii) $\int (x+1)(x^2+3)dx$ (3 marks)

b) Let A and B be sets such that

$$(A \cup B)' = \{x, y, z\}$$

$$B - A = \{w\}$$

$$A \cup B' = \{u, v, t, x, y, z\} \text{ and}$$

$$B \cap A = \{t\}$$

Find A, B and ξ , the smallest universal set for the sets A and B . (6 marks)