(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: MAT 100

COURSE TITLE: MATHEMATICS FOR TECHNOLOGISTS

DATE: 05/10/18   **TIME:** 8 AM -10 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION 1 (30 MARKS)

a) Use the definition of the limit to prove that   (5 marks)

$$\lim_{x \to 2} (5x + 4) = 6$$

b) Find the following limits if they exist:

   i) $\lim_{x \to \infty} \dfrac{2x^4 - x^2 + 8x}{-5x^4 + 7}$ (3 marks)

   ii) $\lim_{x \to 4} \dfrac{x^2 - x - 12}{x - 4}$ (3 marks)

c) Find the derivative of the implicit function

$$7y^2 + \sin(3x) = 12 - y^4$$ (3 marks)

d) Find the equation of the tangent line at the given point.

$$y^2 e^{2x} = 3y + x^2 \quad \text{at} \quad (0,3)$$ (4 marks)

e) Evaluate the following

$$\int x^2 \cos(4x)\, dx$$ (5 marks)

f) Determine the angle between vectors $\mathbf{a} = 2i + 2j + k$ and $\mathbf{b} = 5i + 3j + 2k$ (3 marks)

g) Determine whether the function f given by $f(x) = \dfrac{x^6 - x^2}{x + x^3}$ is even, odd or neither.   (4 marks)

QUESTION 2 (20 MARKS)

a) Find the unit vector that points in the direction of $\mathbf{w} = \langle -5,\ 2,\ 1 \rangle$ (2 marks)

b) Given that $\vec{r} = i - 2k$ and $\vec{r} = i - 2k$ compute each of the following

   i) $\left(\vec{p} - 2\vec{q} \right) \times \vec{r}$ (3 marks)

   ii) $\left| \left(\vec{p} - 2\vec{q} \right) \times \vec{r} \right|$ (2 marks)

c) Prove that $\vec{a} \bullet \vec{b} = \|\vec{a}\| \|\vec{b}\| \cos\theta$ (5 marks)

d) Show that $\mathbf{u} \times \mathbf{u} = 0$ for any vector $\mathbf{u}$.   (3 marks)

e) A function f given by $f(x) = \dfrac{72}{ax + b}$ is such that $f(7)=9$ and $f(6)=12$. Find the values of a and b.

                                                           (5 marks)

QUESTION 3 (20 MARKS)

a) Use the definition of derivative to compute the derivative y' for the function

$$y = 2x^2 - 16x + 35$$
(3 marks)

b) Differentiate each of the following:

    i) $y = \sin\left(3x^2 + x\right)$
(3 marks)

    ii) $y = \dfrac{4\sqrt{x}}{x^2 - 2}$
(3 marks)

c) If $y = 3x^2 + 5x - 10$, show that $y - y'x + \dfrac{1}{2}y''x^2 + 10 = 0$
(4 marks)

d) In a survey of 120 people, it was found that 65 read Newsweek magazine, 45 read Times while 42 read Fortune. If 20 read both Newsweek and Times, 25 read both Newsweek and Fortune, 15 read both Times and Fortune while 8 read all the three magazines:

    i.   Draw a Venn diagram to represent the above information.
(3 marks)

    ii.   Find the number of people who read at least one of the three magazines.
(2 marks)

    iii.   Find the number of people who read exactly one magazine.
(2 marks)

QUESTION 4 (20 MARKS)

a) Compute the determinant of $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 1 & 1 \\ -1 & 3 & 0 \end{pmatrix}$
(5 marks)

b) Show that the matrix $\begin{pmatrix} 1 & 2 & 1 & 1 \\ 2 & 3 & 0 & 5 \\ 3 & 5 & 1 & 4 \end{pmatrix}$ has rank 3.
(2 marks)

c) If $\det A = 2$ and $\det B = 5$, calculate $\det\left(A^3 \, B^{-1} \, A^T \, B^2\right)$.
(5 marks)

d) Determine the particular solution of the differential equation $5\dfrac{dy}{dx} + 2x = 3$, given the boundary conditions $y = 1\dfrac{2}{5}$ when $x = 2$.
(4 marks)

e) The bending moment M of a beam is given by $\dfrac{dm}{dx} = -w\,(l - x)$, where w and l are constants. Determine M in terms of x given that $M = \dfrac{1}{2}wl^2$ when $x = 0$.
(4 marks)

QUESTION 5 (20 MARKS)

a) Determine the following indefinite integrals

    i) $\int 5x \sqrt{(2x^2 + 7)}\, dx$ (6 marks)

    ii) $\int \dfrac{3x + 1}{x^2 - x - 6}$ (5 marks)

    iii) $\int (x + 1)(x^2 + 3)\, dx$ (3 marks)

b) Let A and B be sets such that

$(A \cup B)' = \{x, y, z\}$

$B - A = \{w\}$

$A \cup B' = \{u, v, t, x, y, z\}$ and

$B \cap A = \{t\}$

Find A, B and ξ, the smallest universal set for the sets A and B.   (6 marks)