(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: **MAT 100**

COURSE TITLE: **MATHEMATICS FOR TECHNOLOGISTS**

DATE: 16/01/18   **TIME**: 9 AM -11 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION 1 (30 MARKS)

a) Suppose $A = \{1, 2, 3\}$, $B = \{2, 4, 6\}$ and the Universal set $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, find;

    i) $P(A)$, the power set of A.   (2 marks)

    ii) $A \cup B$ (1 mark)

    iii) $A \cap B$ (1 mark)

    iv) B^c (1 mark)

    v) $A \setminus B$ (1 mark)

b) Find the following limits if they exist:

    i) $\lim\limits_{x \to \infty} \dfrac{2x^4 - x^2 + 8x}{-5x^4 + 7}$ (3 marks)

    ii) $\lim\limits_{x \to 4} \dfrac{x^2 - x - 12}{x - 4}$ (3 marks)

c) Find the derivative of the implicit function

$$7y^2 + \sin(3x) = 12 - y^4$$ (3 marks)

d) Let the functions f and g be defined by $f(x) = 2x + 1$ and $g(x) = x^2 - 2$. Compute;

    i) $f \circ g$ (2 marks)

    ii) $g \circ f(2)$ (3   marks)

e) Evaluate the following

$$\int x^2 \cos(4x)\, dx$$ (4 marks)

f) Determine the angle between vectors $\mathbf{a} = 2i + 2j + k$ and $\mathbf{b} = 5i + 3j + 2k$ (3 marks)

g) Determine whether the function f given by $f(x) = \dfrac{x^6 - x^2}{x + x^3}$ is even, odd or neither.   (3 marks)

QUESTION 2 (20 MARKS)

a) Find the unit vector that points in the direction of $\mathbf{w} = \langle -5, 2, 1 \rangle$ (2 marks)

b) Given that $\vec{r} = i - 2k$ and $\vec{r} = i - 2k$ compute each of the following

    i) $\left(\vec{p} - 2\vec{q} \right) \times \vec{r}$ (3 marks)

    ii) $\left| \left(\vec{p} - 2\vec{q} \right) \times \vec{r} \right|$ (2 marks)

c) Prove that $\vec{a} \bullet \vec{b} = \|\vec{a}\| \|\vec{b}\| \cos\theta$ (5 marks)

d) Show that $\mathbf{u} \times \mathbf{u} = 0$ for any vector $\mathbf{u}$.   (3 marks)

e) A function f given by $f(x) = \dfrac{72}{ax + b}$ is such that $f(7) = 9$ and $f(6) = 12$. Find the values of a and b.   (5 marks)

QUESTION 3 (20 MARKS)

a) Use the definition of derivative to compute the derivative y' for the function

$$y = 2x^2 - 16x + 35$$

(3 marks)

b) Differentiate each of the following:

   i) $y = \sin\left(3x^2 + x\right)$

(3 marks)

   ii) $y = \dfrac{4\sqrt{x}}{x^2 - 2}$

(3 marks)

c) If $y = 3x^2 + 5x - 10$, show that $y - y'x + \dfrac{1}{2}y''x^2 + 10 = 0$

(4 marks)

d) In a survey of 120 people, it was found that 65 read Newsweek magazine, 45 read Times while 42 read Fortune. If 20 read both Newsweek and Times, 25 read both Newsweek and Fortune, 15 read both Times and Fortune while 8 read all the three magazines:

   i.   Draw a Venn diagram to represent the above information. (3 marks)
   ii. Find the number of people who read at least one of the three magazines. (2 marks)
   iii. Find the number of people who read exactly one magazine. (2 marks)

QUESTION 4 (20 MARKS)

a) Compute the determinant of $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 1 & 1 \\ -1 & 3 & 0 \end{pmatrix}$

(3 marks)

b) Show that the matrix $\begin{pmatrix} 1 & 2 & 1 & 1 \\ 2 & 3 & 0 & 5 \\ 3 & 5 & 1 & 4 \end{pmatrix}$ has rank 3.

(2 marks)

c) If $\det A = 2$ and $\det B = 5$, calculate $\det\left(A^3 B^{-1} A^T B^2\right)$.

(5 marks)

d) i) Let $g : x \to \dfrac{1}{2}x$ be a mapping. If the domain of the mapping is $\{0, 1, 2, 3\, 4\}$ list the range of the mapping

(2 marks)

   ii) Find $g^{-1}(x)$ given that $g(x) = \dfrac{2x - 3}{5x - 7}$.

(4 marks)

e) The bending moment M of a beam is given by $\dfrac{dm}{dx} = -w\,(l - x)$, where w and l are constants. Determine M in terms of x given that $M = \dfrac{1}{2}wl^2$ when $x = 0$.

(4 marks)

QUESTION 5 (20 MARKS)

a) Determine the following indefinite integrals

   i) $\int_1^4 (3x+4)^3 \, dx$ (3 marks)

   ii) $\int \dfrac{3x+1}{x^2 - x - 6}$ (5 marks)

   iii) $\int (x+1)(x^2 + 3) \, dx$ (3 marks)

b) Let A and B be sets such that

   $(A \cup B)' = \{x, y, z\}$

   $B - A = \{w\}$

   $A \cup B' = \{u, v, t, x, y, z\}$ and

   $B \cap A = \{t\}$

   Find A, B and ξ, the smallest universal set for the sets A and B.   (6 marks)

c) Write down the subsets of;

   i) $A = \{a\}$ (1 mark)

   ii) $S = \{a, b, c\}$ (2 marks)